CS221 Problem Workout Solutions

Nov 11

1) [CA session] Problem 1

Compute the conjunctive normal form (CNF) of the following two formulas and write every step of your computation:

(a) $\neg P \to \neg\neg(Q \vee (R \wedge \neg S))$

(b) $(P \to (Q \vee (R \wedge S))) \wedge (R \vee (S \to Q))$

Solution

(a)

$\neg P \to \neg\neg(Q \vee (R \wedge \neg S))$ Given

$\neg P \to (Q \vee (R \wedge \neg S))$ Double negation

$\neg\neg P \vee (Q \vee (R \wedge \neg S))$ Implication

$P \vee (Q \vee (R \wedge \neg S))$ Double negation

$(P \vee Q \vee R) \wedge (P \vee Q \vee \neg S)$ Distributivity

(b)

$(P \to (Q \vee (R \wedge S))) \wedge (R \vee (S \to Q))$ Given

$(P \to (Q \vee (R \wedge S))) \wedge (R \vee (\neg S \vee Q))$ Implication

$(\neg P \vee (Q \vee (R \wedge S))) \wedge (R \vee (\neg S \vee Q))$ Implication

$(\neg P \vee Q \vee R) \wedge (\neg P \vee Q \vee S) \wedge (R \vee \neg S \vee Q)$ Distributivity.

2) **[CA session] Problem 2: Proof by Resolution**

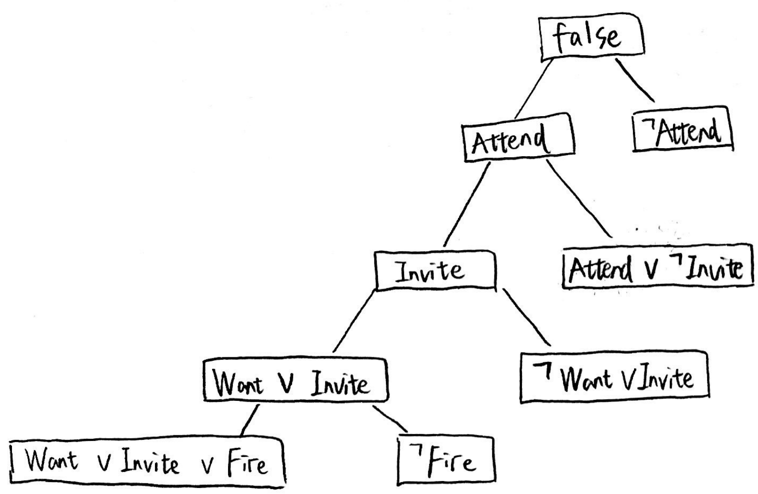
In this question we practice proving by resolution on the following knowledge base: Either Heather attended the meeting or Heather was not invited. If the boss wanted Heather at the meeting, then she was invited. Heather did not attend the meeting. If the boss did not want Heather there, and the boss did not invite her there, then she is going to be fired. Prove Heather is going to be fired.

Solution

$$KB = \{ Attend \vee \neg Invite,\ \neg Want \vee Invite,\ \neg Attend,\ Want \vee Invite \vee Fire \}$$

$$KB' = KB + \text{negation of conclusion}$$

$$= \{ Attend \vee \neg Invite,\ \neg Want \vee Invite,\ \neg Attend,\ Want \vee Invite \vee Fire,\ \neg Fire \}$$



3) [Breakouts] Problem 3

Translate the following English sentences into first-order logic formulas:

(a) Every student takes at least one course.
(b) Every student who takes Analysis also takes Geometry.
(c) No student failed Chemistry but at least one student failed History.

Solution

(a)
$$\forall \text{x (Student(x)} \Rightarrow \exists \text{y (Course(y)} \wedge \text{Takes(x,y)))}$$

(b)
$$\forall \text{x (Student(x)} \wedge \text{Takes(x,Analysis)} \Rightarrow \text{Takes(x,Geometry))}$$

(c)
$$(\neg\exists s(Student(s) \wedge Failed(s,\ Chemistry))) \wedge (\exists x(Student(x) \wedge Failed(x,\ History)))$$