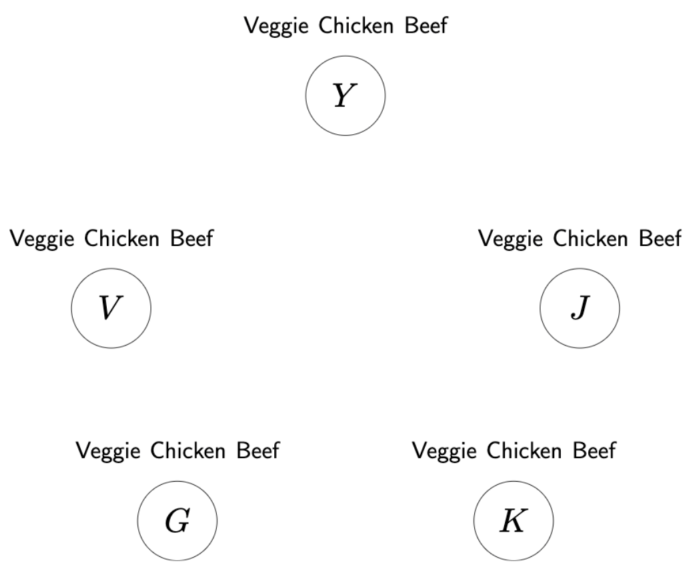
CS221 Problem Workout Solutions

Oct 14

1) [CA session] Problem 1

a. (*10 points*)

You and your friends (Veronica, Jarvis, Gabriela, Kanti) sit around a table like this:



There are three dishes on the menu: the vegetarian deep dish pizza, the chicken quesadilla, and the beef cheeseburger. Each person will order exactly one dish.

But what started out as a simple dinner has quickly turned into a logistical nightmare because of all the constraints you and your friends impose upon yourselves:

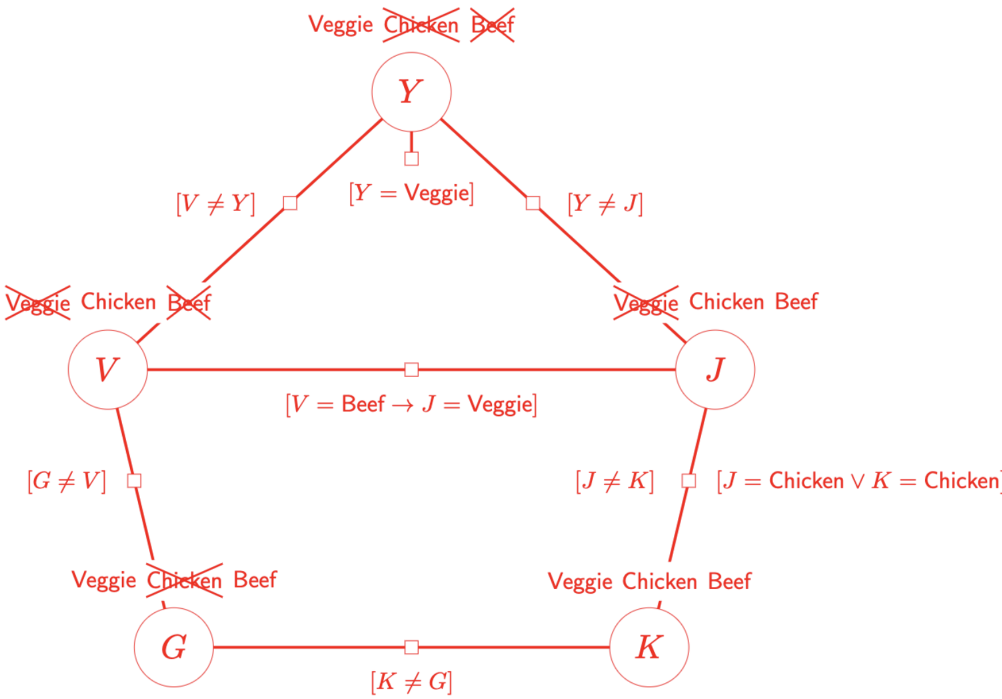
(a) Each person must order something different than the people sitting immediately next to him/her.

(b) You (Y) are vegetarian.

(c) If Veronica (V) orders beef, then Jarvis (J) will order veggie.

(d) Kanti (K) and Jarvis (J) cannot both get non-chicken dishes.

Draw the potentials for the above constraints and write the propositional formula above each potential (e.g., $[Y = \text{Veggie}]$). Then for each pair of variables, enforce arc consistency in both directions, crossing out the appropriate values from the domains.


Veggie Chicken Beef
Y
$[V \neq Y]$
$[Y = \text{Veggie}]$
$[Y \neq J]$
Veggie Chicken Beef
Veggie Chicken Beef
V
J
$[V = \text{Beef} \to J = \text{Veggie}]$
$[G \neq V]$
$[J \neq K]$
$[J = \text{Chicken} \vee K = \text{Chicken}]$
Veggie Chicken Beef
Veggie Chicken Beef
G
K
$[K \neq G]$

Solution

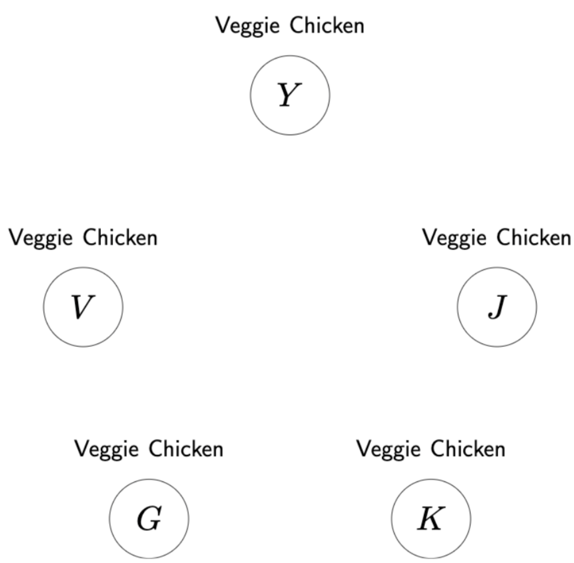
b. (*10 points*)

Your server comes by your table and tells you that they are out of beef today, so you all decide to rework your constraints. Now they are:

(a) There is a preference for people sitting next to each other to order different dishes. Formally, we have 5 potentials: $f(Y, J) = \mathbf{1}[Y \neq J] + 1$, $f(J, K) = \mathbf{1}[J \neq K] + 1$, etc.

(b) You (Y) are vegetarian.

With the 2 constraints above, what is a maximum weight assignment and what is its weight? If there are many assignments with the same maximum weight, give any one. For convenience, the updated table is given below.

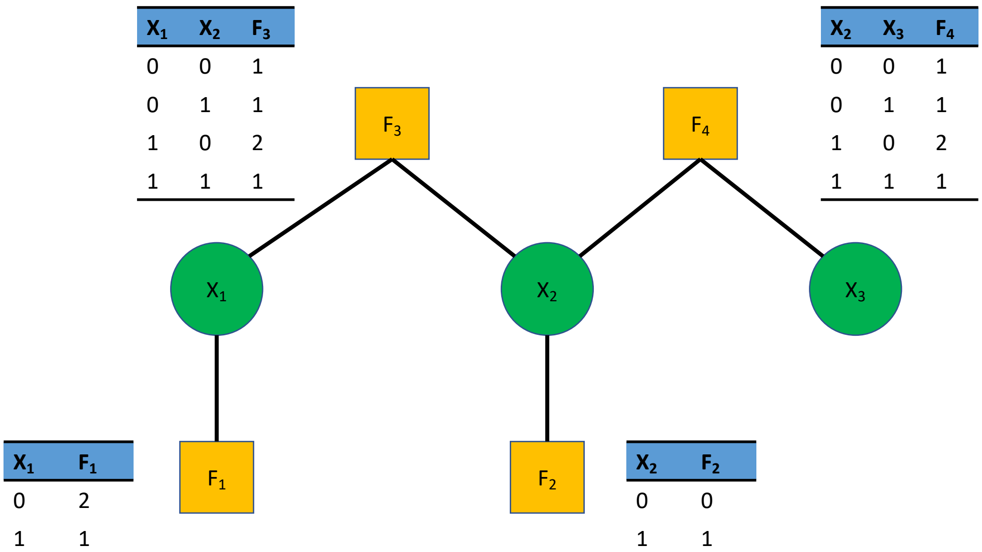


Solution The maximum weight 16. Many answers can be correct. They just need to satisfy the following: 1. Y=Veggie; 2. Only one pair of adjacent people have the same dish. One max weight assignment is:

Y	Veggie
V	Chicken
G	Veggie
K	Chicken
J	Chicken

2) [Breakouts] Problem 2

This problem will give you some practice on computing probabilities given a Markov network. Specifically, given the Markov network below, we will ask you questions about the probability distribution $p(X_1, X_2, X_3)$ over the binary random variables $X_1, X_2,$ and X_3.



X_1	X_2	F_3
0	0	1
0	1	1
1	0	2
1	1	1

X_2	X_3	F_4
0	0	1
0	1	1
1	0	2
1	1	1

X_1	F_1
0	2
1	1

X_2	F_2
0	0
1	1

(a) What is $p(X_1 = 0, X_2 = 0, X_3 = 0)$?

Solution $p(X_1 = 0, X_2 = 0, X_3 = 0) = \frac{F_1(X_1=0)F_2(X_2=0)F_3(X_1=0,X_2=0)F_4(X_2=0,X_3=0)}{Z} = \frac{2\times0\times1\times1}{Z} = 0$, where
$Z = \sum_{X_1\in\{0,1\}} \sum_{X_2\in\{0,1\}} \sum_{X_3\in\{0,1\}} F_1(X_1)F_2(X_2)F_3(X_1, X_2)F_4(X_2, X_3) = 9$

(b) What is $p(X_1 = 0, X_2 = 1, X_3 = 0)$?

Solution $p(X_1 = 0, X_2 = 1, X_3 = 0) = \frac{F_1(X_1=0)F_2(X_2=1)F_3(X_1=0,X_2=1)F_4(X_2=1,X_3=0)}{Z} = \frac{2\times1\times1\times2}{Z} = \frac{4}{9}$, where Z was computed in part a).

(c) What is $p(X_2 = 0)$?

Solution $p(X_2 = 0) = 0$

(d) What is $p(X_3 = 0)$?

Solution $p(X_3 = 0) = \frac{\sum_{X_1 \in \{0,1\}} \sum_{X_2 \in \{0,1\}} p(X_3 = 0, X_1, X_2)}{Z} = \frac{6}{9} = \frac{2}{3}$