CS 221, Fall 2020
Quiz 5 Solutions

1. **[0 points]** You are allowed to consult online lecture notes, personal notes, and use a basic calculator during this quiz. You may not use the internet to access any websites other than Gradescope or the course website. You may not communicate with others to give or receive aid during or after the quiz. **I agree to abide by these rules and the Stanford Honor Code.**

   (a) Agree

   (b) Disagree

   Answer: (a)

2. **[2 points]** Which of the following statements about a **two-player zero-sum game** is **NOT** true?

   (a) The sum of the utility values of the two players is 0 at the end of the game.

   (b) The agent will always have the highest expected utility by applying the minimax algorithm, regardless of the opponent policy.

   (c) Chess is an example of a zero-sum game.

   (d) One player's utility can depend on the other player's actions.

   Answer: (b). If the opponent is playing a stochastic policy π_{opp} and the agent knows exactly what π_{opp} is, the agent will have the highest expected utility by playing the **expectimax** policy.

3. **[2 points]** Samuel and Arlene play the following game: starting with a number N, they take turns either **decrementing** N by 1 or **replacing** it with $\lfloor \frac{N}{2} \rfloor$ (remember that $\lfloor \frac{N}{2} \rfloor$ is defined as the largest integer less than or equal to $\frac{N}{2}$). **The player who reaches 0 on their turn wins**. Now they start from $\mathbf{N = 4}$ and **Samuel moves first**. As an example, if both of them always decrement the number by 1 at each turn, the action sequence would be (Samuel, 4→3), (Arlene, 3→2), (Samuel, 2→1), (Arlene, 1→0). Arlene has 1 at her last turn; after decrementing 1 by 1, she gets 0, so **Arlene wins the game**.
   Now suppose both of them play randomly (meaning that they take either action with the same probability 0.5 at each turn), what's the probability of Samuel winning the game at the end?

   Hint: try to draw the game tree and write down the probability of the corresponding action on each path.

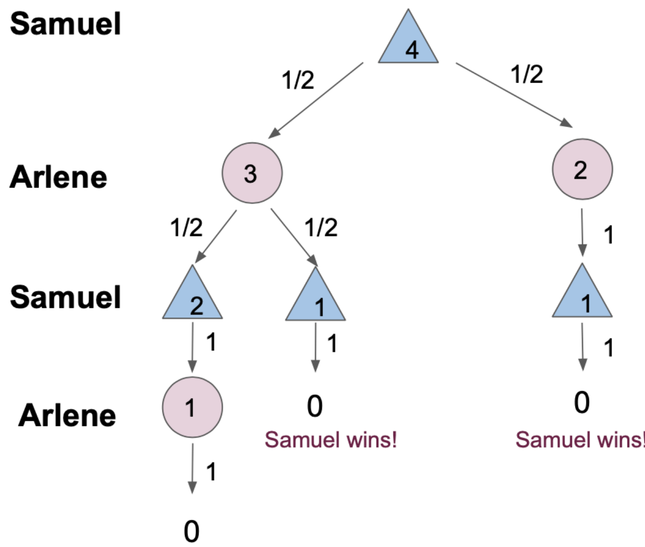
   (a) 1/4

   (b) 1/3

   (c) 3/4

   (d) 2/3

   Answer: (c) See the game tree below. The numbers on the paths indicate the possibilities of the corresponding actions. There are two possible sequences leading to Samuel winning; their possibilities are respectively: $\frac{1}{2} * \frac{1}{2} * 1 = \frac{1}{4}$ and $\frac{1}{2} * 1 * 1 = \frac{1}{2}$. Thus, the total possibility is $\frac{1}{4} + \frac{1}{2} = \frac{3}{4}$.



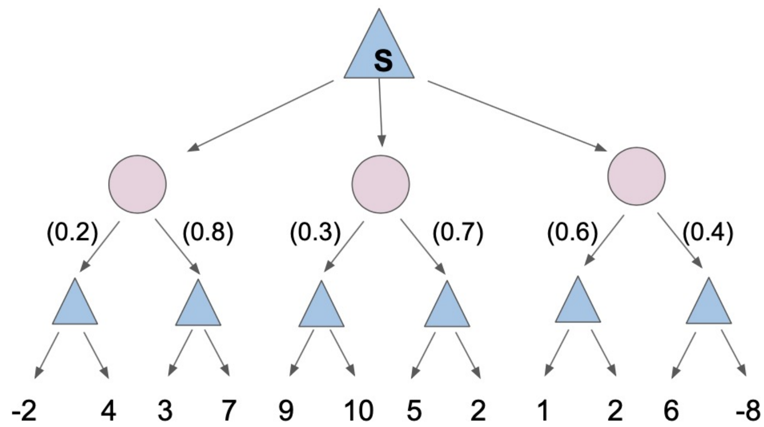
4. **[2 points]** In a two-player zero-sum game, if the opponent is known to be playing a **stochastic** policy π_{opp} and the agent knows exactly what π_{opp} is, the agent will always have the **highest expected utility** by following the **minimax** policy.

   (a) True

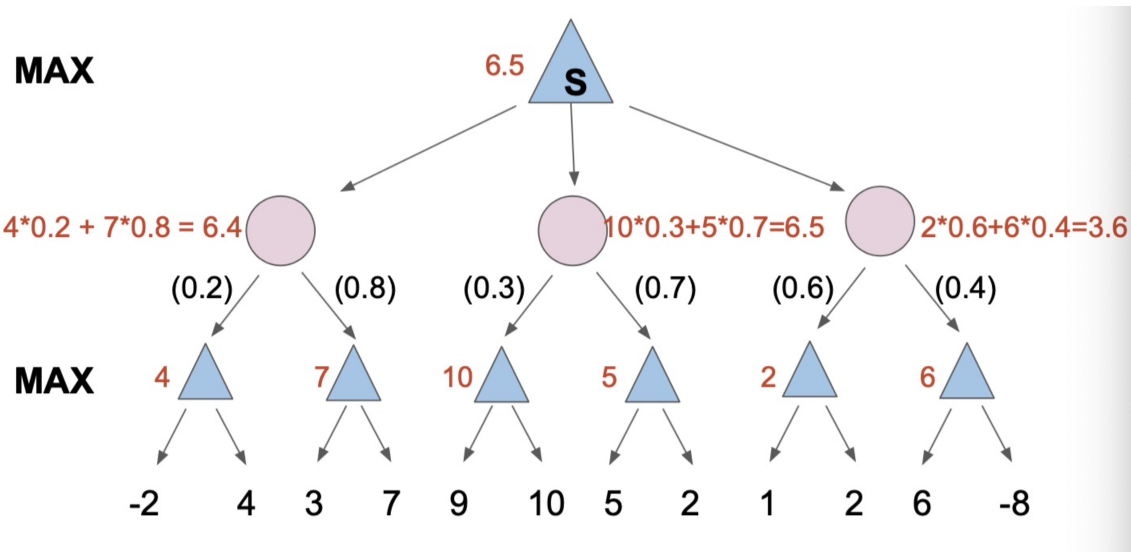
   (b) False

   Answer: (b) If the opponent is known to be playing a stochastic policy π_{opp}, the agent will have the highest expected utility by following the **expectimax** policy.

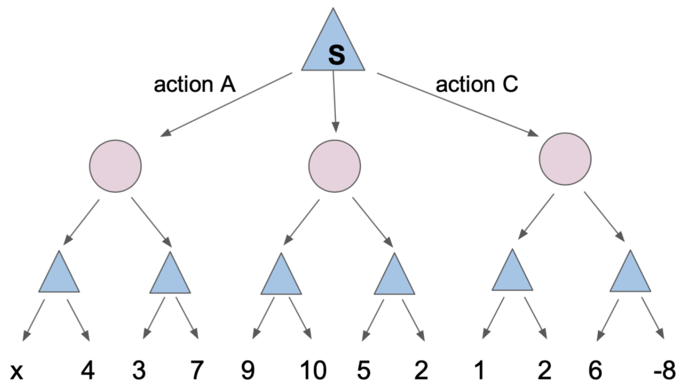
5. **[2 points]** Consider the following game tree, where the triangles are states controlled by the agent (i.e., it's the agent's turn to take actions at these states) and the circles are states controlled by the opponent. S is the start state. The numbers on the leaves are utility values for the agent at the end states. Suppose we know that the opponent plays a stochastic policy, and the probabilities of the opponent taking various actions are indicated on the corresponding paths. Run the **expectimax algorithm** to calculate $V_{exptmax}(S)$ for start state S. Write down the value of $V_{exptmax}(S)$.



Answer: $V_{mimax}(S) = 6.5$, as the following tree shows.



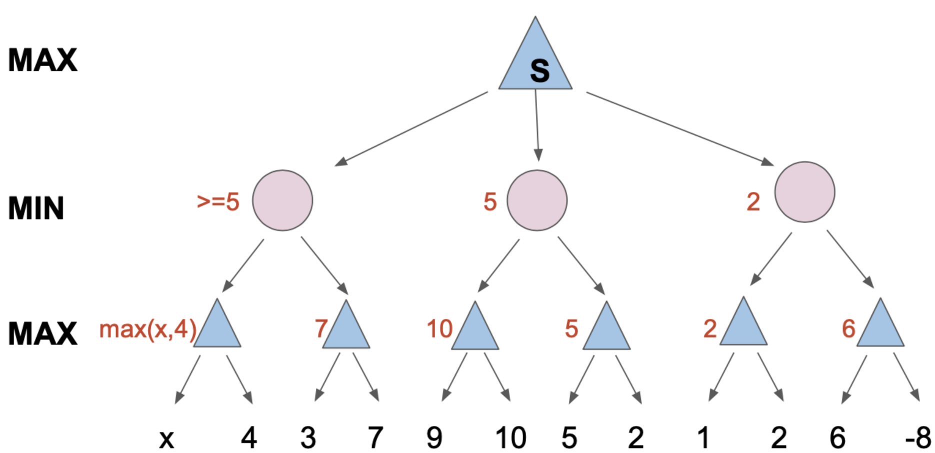
6. **[3 points]** Consider the following modified game tree, where the leftmost leaf has an unknown utility value x. The agent moves first, and attempts to maximize the utility. The agent and opponent are following minimax policies π_{max} and π_{min} respectively.



(i) For what values of x does the agent choose the leftmost action (action A, as shown on the figure) as the first step? Assume that in case of a tie, the agent always picks the leftmost action in the tie.

(a) $x < 5$
(b) $x \geq 5$
(c) $x < 2$
(d) $x \geq 2$
(e) The agent will never choose action A, regardless of the value of x.

Answer: (b)
See the figure below. In order for the agent to choose the leftmost action, we must have the value of the leftmost Min node ≥ 5, or equivalently $min(7, max(x, 4)) \geq 5$. Thus $max(x, 4) \geq 5, x \geq 5$.



(ii) For what values of x does the agent choose the rightmost action (action C, as shown on the figure) as the first step? Assume that in case of a tie, the agent always picks the leftmost action in the tie.

(a) $x < 5$
(b) $x \geq 5$
(c) $x < 2$
(d) $x \geq 2$
(e) The agent will never choose action C, regardless of the value of x.

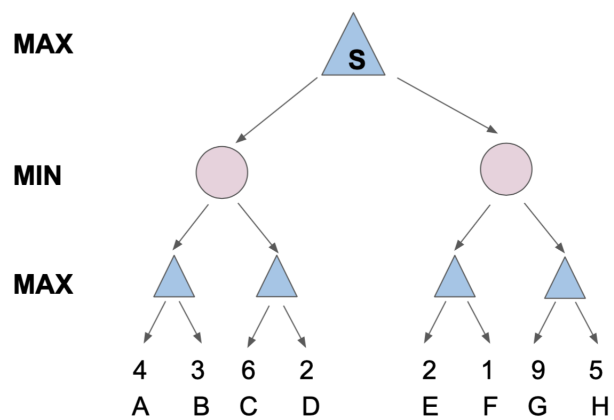
Answer: (e) See the figure from part (i) solution. The rightmost Min node has value 2, which is smaller than the value of the Min node in the middle (5), so the agent will always prefer the action in the middle than the rightmost action. Additionally, if $x \geq 5$, the agent will prefer the leftmost action than the action in the middle.

7. **[2 points]** Using alpha-beta pruning may return a worse move than minimax search since it explores fewer branches.

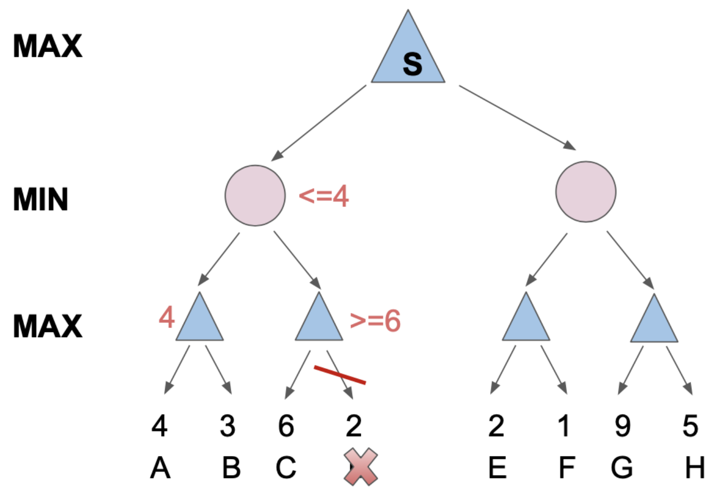
(a) True
(b) False

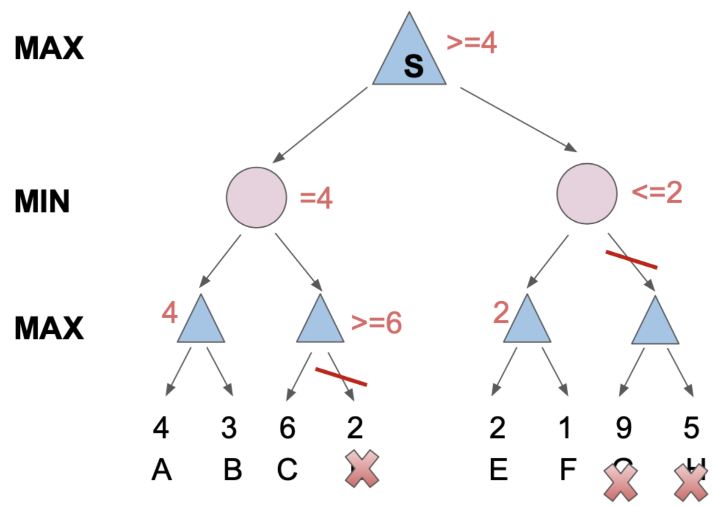
Answer: (b) Alpha-beta pruning will return exactly the same solution as the minimax algorithm.

8. **[2 points]** Consider applying alpha-beta pruning to the following game tree. The children of each node are expanded from left to right. Write down the **leaf nodes** that will **not** be examined because they are **pruned** by alpha-beta pruning, in the order left to right. For example, if the leaf nodes A, B and C are pruned, the answer should be "A,B,C".



Answer: "D,G,H".
See the following figures. The first figure illustrates why we prune D, and the second one illustrates why we prune G and H.


MAX
S
MIN
<=4
MAX
4
>=6
4 3 6 2 2 1 9 5
A B C E F G H


MAX
S
>=4
MIN
=4
<=2
MAX
4
>=6
2
4 3 6 2 2 1 9 5
A B C E F