CS 221, Fall 2020
Quiz 6 Solutions

1. **[0 points]** You are allowed to consult online lecture notes, personal notes, and use a basic calculator during this quiz. You may not use the internet to access any websites other than Gradescope or the course website. You may not communicate with others to give or receive aid during or after the quiz. **I agree to abide by these rules and the Stanford Honor Code.**

   (a) Agree

   (b) Disagree

   Answer: (a)

2. **[2 points]** How many factors must evaluate to zero in a constraint satisfaction problem to make an assignment inconsistent?

   (a) None

   (b) At least 1

   (c) At least 2

   (d) At least 3

   Answer: (b) See slide 14: https://stanford-cs221.github.io/autumn2020/modules/module.html#include=csps/definitions.js&mode=print1pp

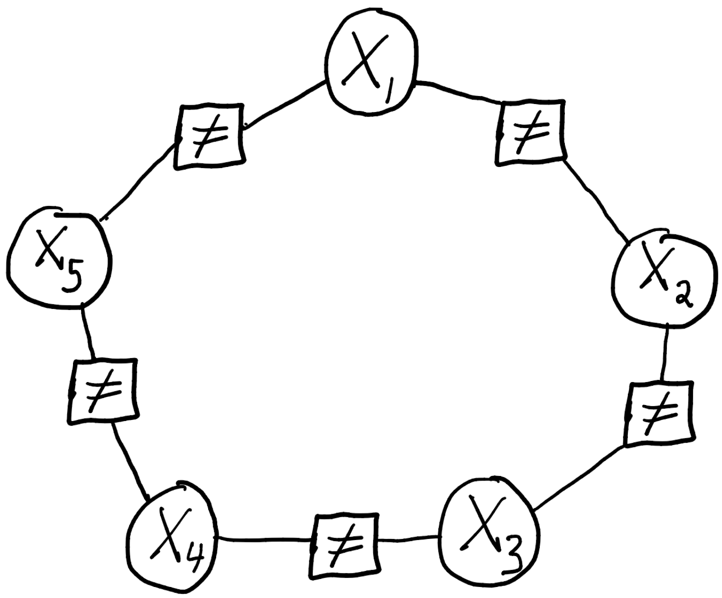
3. **[2 points]** If beam search is run with $K = 100$ and given unlimited runtime, then it is guaranteed to find the 100 highest weight assignments in any constraint satisfaction problem.

   (a) True

   (b) False

   Answer: (b) See slide 14: https://stanford-cs221.github.io/autumn2020/modules/module.html#include=csps/beam-search.js&mode=print1pp

4. **[12 points] Disagreeable Dining** Five friends (X_1, X_2, X_3, X_4, X_5) are going out for dinner, but on the way disagreements break out. They decide to split up, with each friend eating at either restaurant A or B. X_1 will not eat at the same restaurant as X_2, X_2 will not eat at the same restaurant as X_3, X_3 will not eat at the same restaurant as X_4, X_4 will not eat at the same restaurant as X_5, and X_5 will not eat at the same restaurant as X_1. This situation can be modeled as the constraint satisfaction problem shown in Figure 5. The domain of each variable is $\{A, B\}$. Factors take the value 0 if any two variables within their scope take the same value, 1 otherwise.



(a) **[2 points]** This constraint satisfaction problem is satisfiable.

(a) True

(b) False

Answer: (b) See slide 14: https://stanford-cs221.github.io/autumn2020/modules/module.html#include=csps/definitions.js&mode=print1pp

(b) **[2 points]** Say we start with the empty assignment, and then run AC-3 with $S \leftarrow \{X_1\}$, as presented in slide 10 of https://stanford-cs221.github.io/autumn2020/modules/module.html#include=csps/arc-consistency.js&mode=print1pp. Will AC-3 determine that the CSP has **no** consistent assignments?

(a) Yes

(b) No

Answer: (b) Running AC-3 in this setup will not remove values from any variable domains because $\text{Domain}_1 = \{A, B\}$.

(c) **[2 points]** If X_1 is assigned the value A, what are the updated domains of all other variables after running one round of forward checking (i.e., eliminating inconsistent values from the domains of X_1's neighbors)?

Answer: $\text{Domain}_2 = \{B\}, \text{Domain}_3 = \{A, B\}, \text{Domain}_4 = \{A, B\}, \text{Domain}_5 = \{B\}$

(d) **[2 points]** If X_1 is assigned the value A, and we run AC-3 with $S \leftarrow \{X_1\}$ as before, will AC-3 determine that the CSP has **no** consistent assignments?

(a) Yes

(b) No

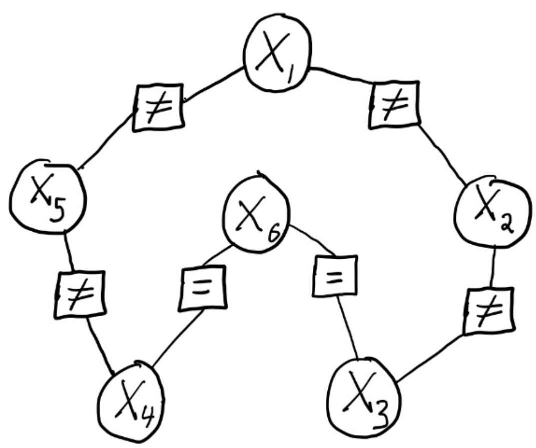
Answer: (a) see slide 10: https://stanford-cs221.github.io/autumn2020/modules/module.html#include=csps/arc-consistency.js&mode=print1pp

(e) **[2 points]** Friends X_3 and X_4 resolve their disagreement and no longer refuse to dine at the same restaurant. A sixth friend (X_6) is also invited to dinner, whose only constraints are that they must dine at the same restaurant as both X_3 and X_4. Draw a constraint satisfaction problem corresponding to the new situation (don't forget that we still need to consider X_1, X_2, and X_5). Is the new constraint satisfaction problem satisfiable?

(a) Yes

(b) No

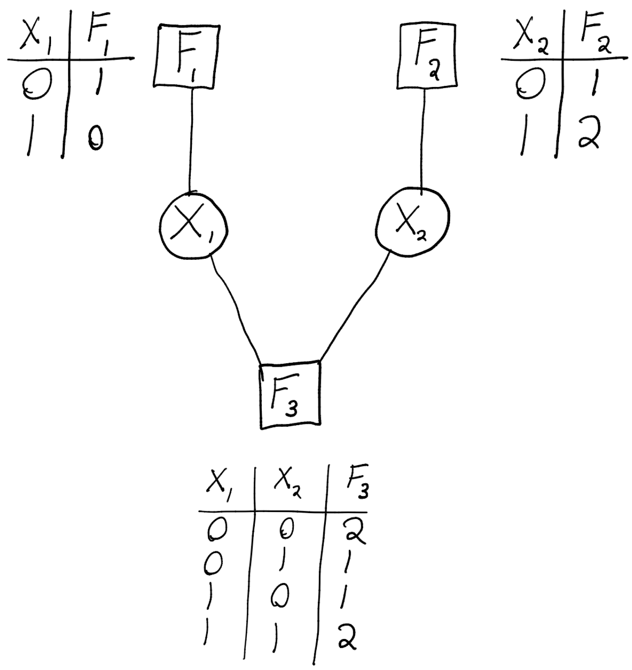
Answer: (a) $X_1 = A, X_2 = B, X_3 = A, X_6 = A, X_4 = A, X_5 = B$ is a satisfying solution. Modified factor graph below (equality factors take the value 1 if the variables within their scope all take the same value, otherwise equality factors take the value 0).



(f) **[2 points]** If X_1 is assigned the value A in the **factor graph you drew for question 4.5**, what are the updated domains of all other variables after running AC-3 with $S \leftarrow \{X_1\}$?

Answer: $\text{Domain}_2 = \{B\}, \text{Domain}_3 = \{A\}, \text{Domain}_4 = \{A\}, \text{Domain}_5 = \{B\}, \text{Domain}_6 = \{A\}$

5. **[7 points]** The following questions concern the probability distribution $p(X_1, X_2)$ over the binary random variables X_1 and X_2 represented by the following Markov network.



(a) **[1 points]** What is $P(X_1 = 1, X_2 = 1)$?
Answer: $P(X_1 = 1, X_2 = 1) = \frac{F_1(X_1=1)F_2(X_2=1)F_3(X_1=1,X_2=1)}{Z} = \frac{0\times 2\times 2}{Z} = 0$, where $Z = \sum_{x_1\in 0,1}\sum_{x_2\in 0,1} F_1(X_1 = x_1)F_2(X_2 = x_2)F_3(X_1 = x_1, X_2 = x_2) = 4$

(b) **[1 points]** What is $P(X_1 = 1)$?
Answer: $P(X_1 = 1) = \sum_{x_2\in 0,1} \frac{F_1(X_1=1)F_2(X_2=x_2)F_3(X_1=1,X_2=x_2)}{Z} = 0$, where $Z = \sum_{x_1\in 0,1}\sum_{x_2\in 0,1} F_1(X_1 = x_1)F_2(X_2 = x_2)F_3(X_1 = x_1, X_2 = x_2) = 4$

(c) **[1 points]** What is $P(X_2 = 1)$?
Answer: $P(X_2 = 1) = \sum_{x_1\in 0,1} \frac{F_1(X_1=x_1)F_2(X_2=1)F_3(X_1=x_1,X_2=1)}{Z} = .5$, where $Z = \sum_{x_1\in 0,1}\sum_{x_2\in 0,1} F_1(X_1 = x_1)F_2(X_2 = x_2)F_3(X_1 = x_1, X_2 = x_2) = 4$

(d) **[2 points]** Say we try using Gibbs sampling to estimate the marginal $P(X_2 = 0)$. We start by arbitrarily initializing $X_1 = 0$ and $X_2 = 0$, and then run one loop of Gibbs sampling: first, we sample X_1 given $X_2 = 0$, then we sample X_2 given the new sampled value of X_1. What is the probability that we sample $X_2 = 0$ at the end of this first loop, given this initialization of $X_1 = 0$ and $X_2 = 0$?
Answer: 0.5
The probability that we sample $X_0 = 0$ at the end of the first loop is

$$\sum_{x_1\in\{0,1\}} P(X_1 = x_1|X_2 = 0)P(X_2 = 0|X_1 = x_1) = .5 \tag{1}$$

($P(X_1 = 0|X_2 = 0) = 1$ and $P(X_2 = 0|X_1 = 0) = .5$)

(e) **[2 points]** Would the above answer change if we instead started with an initialization of $X_1 = 1$ and $X_2 = 0$?

(a) No. The probability of sampling $X_2 = 0$ at the end of the first loop will be the same.
(b) Yes. The probability of sampling $X_2 = 0$ at the end of the first loop will be different, even though the estimated marginal $\hat{P}(X_2 = 0)$ will converge to the same value after running enough loops of Gibbs sampling.
(c) Maybe. The behavior of Gibbs sampling in this case will be undefined, because the initial assignment has zero probability.

Answer: (a) No, the initialization of X_1 does not change the answer since we begin by sampling a new value for X_1.