CS 221, Fall 2020
Quiz 8 Solutions

1. **[0 points]** You are allowed to consult online lecture notes, personal notes, and use a basic calculator during this quiz. You may not use the internet to access any websites other than Gradescope or the course website. You may not communicate with others to give or receive aid during or after the quiz. **I agree to abide by these rules and the Stanford Honor Code.**

   (a) Agree

   (b) Disagree

   Answer: (a)

2. Consider a setting with four propositional symbols: Healthy, RunnyNose, Flu, and Fever. Let your current knowledge base be

   $\text{KB} = \{\text{RunnyNose} \rightarrow \neg\text{Flu}, \text{Fever} \wedge \text{Flu}\}$.

   You are asked by the CDC to update the knowledge base with some new formulas, but the CDC would like to know how the new formulas will change the set of models representing the knowledge base.

   (a) **[1 points]** First, the CDC would like to know if the knowledge base is satisfiable. Select the True statement.

      (a) KB is satisfiable

      (b) KB is **not** satisfiable.

      Answer: A. Take the assignment {RunnyNose: 0, Fever: 1, Flu: 1 }. We can see this is a model in the KB, so the KB is satisfiable.

   (b) **[2 points]** CDC researcher Arnette gives you the following new formula to compare with your knowledge base: $f_1 = \text{RunnyNose} \vee \text{Healthy}$. Select the True statement.

      (a) KB entails f_1.

      (b) KB contradicts f_1 .

      (c) f_1 is contingent.

      Answer: C. First, we check that $KB \cup \{\neg f_1\}$ is satisfiable, which it is. Then, we check that $KB \cup \{f_1\}$ is satisfiable, which it is, so f_1 is contingent. See slide 36 of propositional-logic-semantics for more details.

   (c) **[2 points]** CDC researcher Bastille gives you the following new formula to compare with your knowledge base: $f_2 = \text{RunnyNose} \wedge \text{Flu}$.

      (a) KB entails f_2.

      (b) KB contradicts f_2 .

      (c) f_2 is contingent.

      Answer: B. We can see that $KB \cup \{f_2\}$ is not satisfiable, so KB contradicts f_2.

3. **[2 points]** The CDC gives you a new knowledge base

   $\text{KB} = \{\neg\text{Fever} \rightarrow \text{Healthy},$
   $(\neg\text{Flu} \wedge \neg\text{Fever}) \rightarrow \neg\text{RunnyNose},$
   $\neg\text{Fever},$
   $\text{Healthy} \rightarrow \neg\text{Flu}\}$.

   Perform forward inference using only the modus ponens inference rule and list all formulas added to the knowledge base. Please use the generalized modus ponens inference rule that can take in an arbitrary number of premises. Please use the the minus sign to represent negation and add spaces between your comma-separated answers.

   Example answer format: -Fever, Healthy

   Answer: $-Flu, Healthy, -RunnyNose$

See slide 6 of propositional-modus-ponens for more details. First, we can use modus ponens to derive $Healthy$ since we know $\neg Fever$. Then, we can derive $\neg Flu$ since we know $Healthy$. Finally, we can derive $\neg RunnyNose$ since we know $\neg Flu$ and $\neg Fever$.

4. **[1 points]** You are given a knowledge base

   KB = $\{A,$
   $A \wedge B,$
   $A \wedge B \rightarrow D,$
   $D \wedge A \wedge B \rightarrow C,$
   $E \rightarrow \text{false}\}.$

   Select the True statement from below.

   (a) Modus ponens is sound on the given KB, but not necessarily complete.
   (b) Modus ponens is complete on the given KB, but not necessarily sound.
   (c) Modus ponens is sound and complete on the given KB.
   (d) Modus ponens is neither sound nor complete on the given KB.

   Answer: C. Modus ponens is sound and complete for propositional logic with horn clauses: see propositional-modus-ponens slides.

   Although $A \wedge B$ is not technically a horn clause, it can be split into A and B separately in the knowledge base, in which case the KB would indeed only contain horn clauses. Because of this ambiguity, we accept answers (a) or (c).

5. **[2 points]** Select the correct conversion of the following formula to conjunctive normal form (CNF): $(A \wedge B) \rightarrow (C \wedge D)$.

   (a) $(\neg A \vee \neg B) \vee (C \wedge D)$
   (b) $(\neg A \vee \neg B \vee C) \wedge (\neg A \vee \neg B \vee D)$
   (c) $(A \vee B \vee C) \wedge (A \vee B \vee D)$
   (d) $(\neg A \vee \neg B) \wedge (C \vee D)$

   Answer: B. First, we remove the implication and get $\neg(A \wedge B) \vee (C \wedge D)$. We then push in the negation to get $(\neg A \vee \neg B) \vee (C \wedge D)$. Then we distribute the $\vee$ over the $\wedge$ to obtain the final answer of B. See slide 14 of propositional-resolution for more details.

6. **[2 points]** Your friend can only read first order logic and needs your help to convert an English sentence into first order logic. Let us consider the following formulas: *Student(x)* means x is a student, *Classroom(x)* means x is a classroom, *Safe(x)* means x is safe, and *Wears(x, mask)* means x wears a mask. Select the correct conversion of the following English sentence into a first order logic formula: *None of the classrooms are safe if not all students wear masks.*

   [Note: There may be multiple ways to convert this sentence to first order logic, but only one of the answer choices is a correct conversion]:

   (a) $(\forall s\ \textit{Student(s)} \wedge \rightarrow Wears(s, mask)) \rightarrow (\forall c\ \textit{Classroom(c)} \rightarrow \neg Safe(c))$
   (b) $(\exists s\ \textit{Student(s)}\ \wedge \neg Wears(s, mask)) \rightarrow (\exists c\ \textit{Classroom(c)} \wedge \neg Safe(c))$
   (c) $(\exists s\ \textit{Student(s)}\ \rightarrow \neg Wears(s, mask)) \rightarrow (\forall c\ \textit{Classroom(c)} \wedge \neg Safe(c))$
   (d) $(\exists s\ \textit{Student(s)}\ \wedge \neg Wears(s, mask)) \rightarrow (\forall c\ \textit{Classroom(c)} \rightarrow \neg Safe(c))$

   Answer: D. We can rephrase the statement as: if there exists a student that does not wear a mask, then all classrooms are not safe, which is translated in answer choice D.