

CS221 Problem Workout

Week 5

1) [CA session] Problem 1

After finally meeting up, Romeo (R) and Juliet (J) decide to try to catch a goose (G) to keep as a pet. Eventually, they chase it into a 3×3 hedge maze show below. Now they play the following turn-based game:

- (a) The Goose moves either Down or Right.
- (b) Romeo moves either Up or Right.
- (c) Juliet moves either Left or Down.

G	o	J
	WALL	
R		

Participants: Goose (G), Romeo (R), Juliet (J), bread (o)

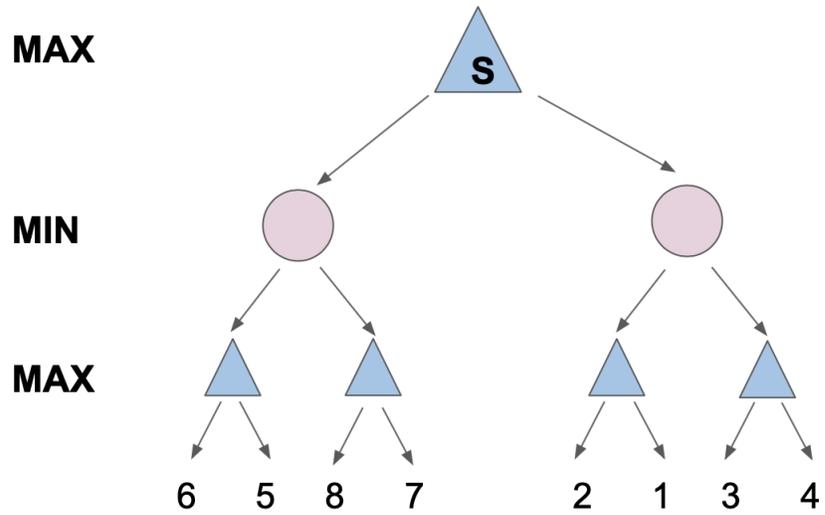
If the Goose enters the square with bread, it gets a reward 1. If either Romeo or Juliet enters the same square as the Goose, they catch it and the Goose gets a reward of -50 . The game ends when either the Goose has been caught or everyone has moved once. Note that it is possible for the Goose to get both rewards.

Construct a depth one minimax tree for the above situation, with the Goose as the maximizer and Juliet and Romeo as the minimizers. Use up-triangles Δ for max nodes, down-triangles ∇ for min nodes, and square nodes for the leaves. Label each node with its minimax value.

What is the minimax value of the game if Romeo defects and becomes a maximizer?

2) [CA session] Problem 2

Consider running alpha-beta pruning on the following minimax tree. The children of each node will be expanded from left to right. Which nodes will be pruned (thus not being visited)?



3) [Breakouts] Problem 3

Consider the speed bump problem we did last week:

You're programming a self-driving car that can take you from home (position 1) to school (position n). At each time step, the car has a current position $x \in \{1, \dots, n\}$ and a current velocity $v \in \{0, \dots, m\}$. The car starts with $v = 0$, and at each time step, the car can either increase the velocity by 1, decrease it by 1, or keep it the same; this new velocity is used to advance x to the new position. The velocity is not allowed to exceed the speed limit m nor return to 0.

In addition, to prevent people from recklessly cruising down Serra Mall, the university has installed speed bumps at a subset of the n locations. The speed bumps are located at $B \subseteq \{1, \dots, n\}$. The car is not allowed to enter, leave, or pass over a speed bump with velocity more than $k \in \{1, \dots, m\}$. **Your goal is to arrive at position n with velocity 1 in the smallest number of time steps.**

Now let's add more information to this problem:

The university wants to remove the old speed bumps and install a single new speed bump at location $b \in \{1, \dots, n\}$ to maximize the time it takes for the car to go from position 1 to n .

Let $T(\pi, B)$ be the time it takes to get from 1 to n if the car follows policy π if speed bumps B are present. If π violates the speed limit, define $T(\pi, B) = \infty$.

To simplify, assume $n = 6$ and $k = 1$. Again, there is exactly one speed bump. That is, $B = \{b\}$ with $b \in \{1, \dots, n\}$.

$x = 1$ home	$x = 2$	$x = 3$	$x = 4$	$x = 5$	$x = 6$ school
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Figure: The university will add a speed bump somewhere

(i) [5 points] Compute the worst case driving time, assuming you get to adapt your policy to the university's choice of speed bump location b : $\max_b \min_{\pi} T(\pi, \{b\})$. What values of b attain the maximum?

(ii) [5 points] Compute the best possible time assuming that you have to choose your policy before the university chooses the speed bump: $\min_{\pi} \max_b T(\pi, \{b\})$. Make sure to explain your reasoning.