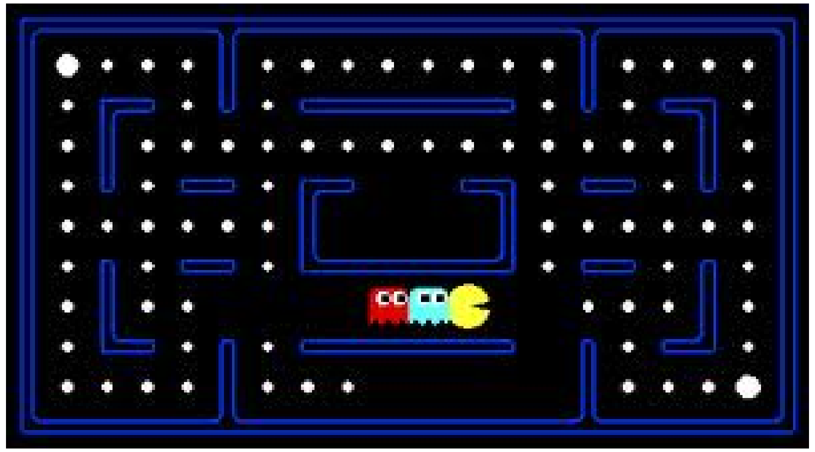
Games: expectiminimax



A modified game

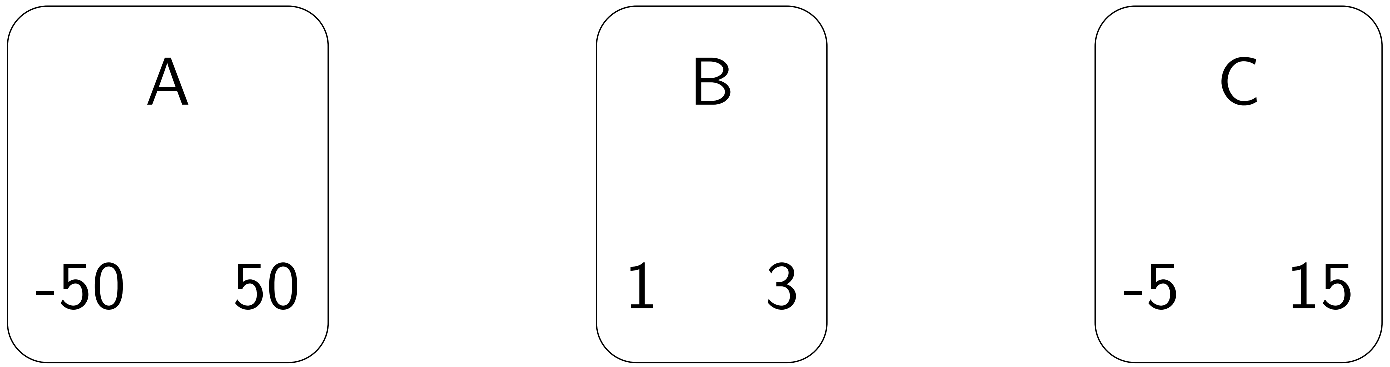
Example: game 2

You choose one of the three bins.

Flip a coin; if heads, then move one bin to the left (with wrap around).

I choose a number from that bin.

Your goal is to maximize the chosen number.

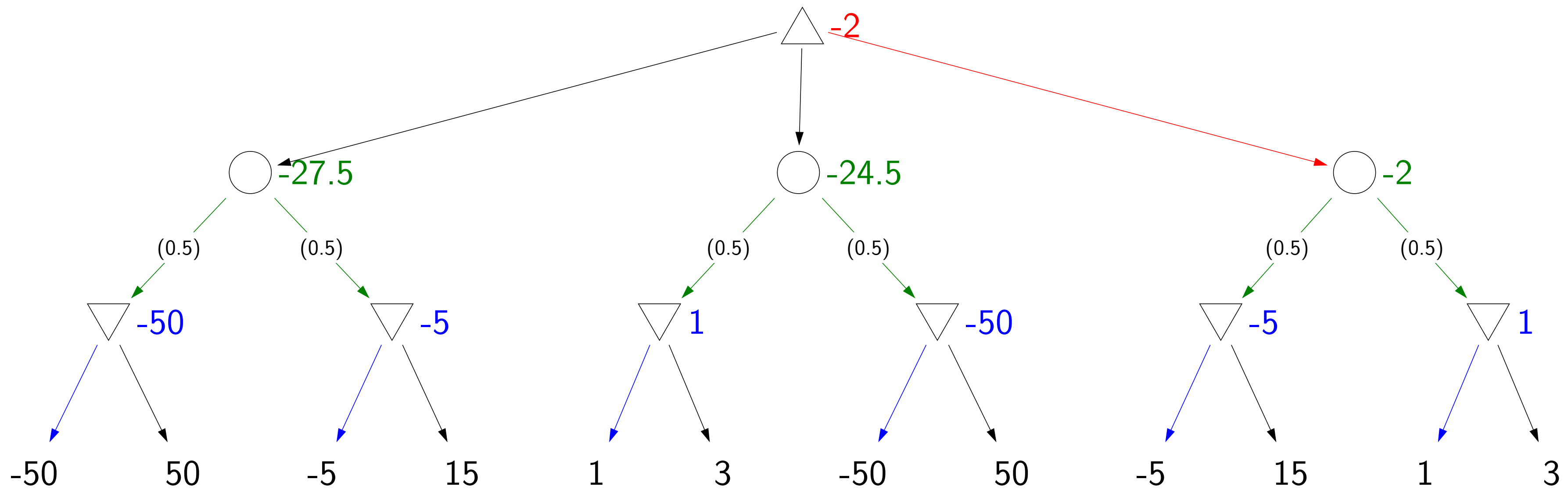


- Now let us consider games that have an element of chance that does not come from the agent or the opponent. Or in the simple modified game, the agent picks, a coin is flipped, and then the opponent picks.
- It turns out that handling games of chance is just a straightforward extension of the game framework that we have already.

Expectiminimax example

Example: expectiminimax

$\pi_{\text{coin}}(s, a) = \frac{1}{2}$ for $a \in \{0, 1\}$

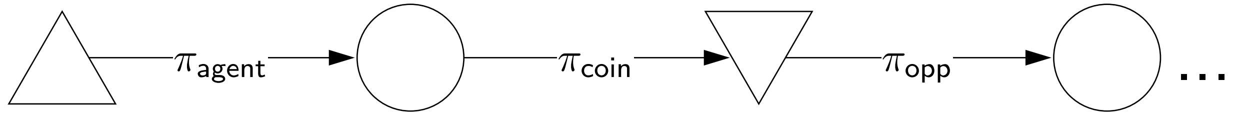


$$\boxed{V_{\text{exptminmax}}(s_{\text{start}}) = -2}$$

- In the example, notice that the minimax optimal policy has shifted from the middle action to the rightmost action, which guards against the effects of the randomness. The agent really wants to avoid ending up on A, in which case the opponent could deliver a deadly -50 utility.

Expectiminimax recurrence

$\text{Players} = \{\text{agent}, \text{opp}, \text{coin}\}$



$$V_{\text{exptminmax}}(s) = \begin{cases} \text{Utility}(s) & \text{IsEnd}(s) \\ \max_{a \in \text{Actions}(s)} V_{\text{exptminmax}}(\text{Succ}(s,a)) & \text{Player}(s) = \text{agent} \\ \min_{a \in \text{Actions}(s)} V_{\text{exptminmax}}(\text{Succ}(s,a)) & \text{Player}(s) = \text{opp} \\ \sum_{a \in \text{Actions}(s)} \pi_{\text{coin}}(s,a) V_{\text{exptminmax}}(\text{Succ}(s,a)) & \text{Player}(s) = \text{coin} \end{cases}$$

- The resulting game is modeled using **expectiminimax**, where we introduce a third player (called coin), which always follows a known stochastic policy. We are using the term *coin* as just a metaphor for any sort of natural randomness.
- To handle coin, we simply add a line into our recurrence that sums over actions when it's coin's turn.

Summary so far

Primitives: max nodes, chance nodes, min nodes

Composition: alternate nodes according to model of game

Value function $V_{\ldots}(s)$: recurrence for expected utility

Scenarios to think about:

- What if you are playing against multiple opponents?
- What if you and your partner have to take turns (table tennis)?
- Some actions allow you to take an extra turn?

- In summary, so far, we've shown how to model a number of games using game trees, where each node of the game tree is either a max, chance, or min node depending on whose turn it is at that node and what we believe about that player's policy.
- Using these primitives, one can model more complex turn-taking games involving multiple players with heterogeneous strategies and where the turn-taking doesn't have to strictly alternate. The only restriction is that there are two parties: one that seeks to maximize utility and the other that seeks to minimize utility, along with other players who have known fixed policies (like coin).