# Basics: Linear Algebra

**Vector**: tuple of real numbers (a.k.a scalars)

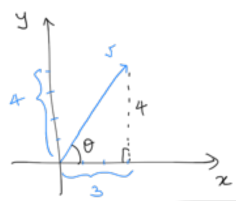
$v = (v_1, v_2 \dots v_d)$

components of vector $v$

$v \in \mathbb{R}^d$

**Example ($d=2$)**

$v = (3, 4) \quad v \in \mathbb{R}^2$




magnitude + direction

length of $v = \sqrt{3^2 + 4^2} = 5$

$\tan\theta = \dfrac{v_2}{v_1} = \dfrac{4}{3}$

**Norm of a vector**

$v \in \mathbb{R}^d \qquad \|v\| \in \mathbb{R}$

$$\|v\| = \sqrt{v_1^2 + v_2^2 \dots v_d^2}$$

$$\boxed{\|v\| = \sqrt{\sum_{i=1}^{d} v_i^2}}$$

$l_2$ norm

**Normalization of a vector**

$\hat{v}$ : normalization of $v$

(i) unit norm
(ii) direction unchanged

$$\hat{v} = \left(\frac{v_1}{\|v\|}, \frac{v_2}{\|v\|} \dots \frac{v_d}{\|v\|}\right)$$

$\|\hat{v}\| = 1$

**Operations on vectors**

① Vector addition: $a, b \in \mathbb{R}^d$

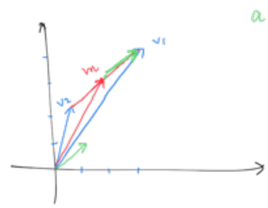
$c = a + b \quad c \in \mathbb{R}^d$

$c_i = a_i + b_i \qquad c = (a_1+b_1, a_2+b_2 \dots a_d+b_d)$

Aside: $v_1 = (3,4) \quad v_2 = (1,2)$

"centroid" $= m = \dfrac{v_1+v_2}{2} = (2,3)$

$a = v_1 - m$
$= (1, 1)$




② Scalar multiplication: $\alpha \in \mathbb{R} \quad v \in \mathbb{R}^d$

$c = \alpha v \quad c \in \mathbb{R}^d$

$c_i = \alpha \cdot v_i$ $\quad c = (v_1\alpha,\ v_2\alpha \dots v_d\alpha)$

(i) direction unchanged

(ii) $\hat{v} = \left(\frac{1}{\|v\|}\right)\cdot v$

③ <u>Dot product</u> $\quad a, b \in \mathbb{R}^d \quad f \in \mathbb{R}$

<u>Notation</u>: $a \bullet b$, $b \bullet a$.

$a^T b$, $b^T a$

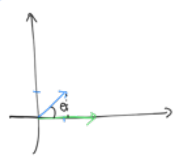
$$f = a \bullet b = a_1 b_1 + a_2 b_2 \dots a_d b_d$$

$$\boxed{f = \sum_{i=1}^{d} a_i b_i}$$

<u>Examples (d=2)</u>

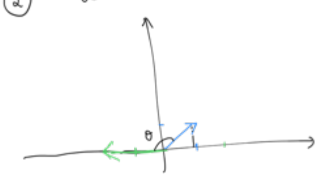
① $a = (1,1)$ $\quad b = (2,0)$

$f = a \bullet b = 2$



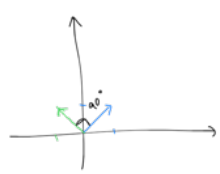
② $a = (1,1)$ $\quad b = (-2,0)$

$f = a \bullet b = -2$



③ $a = (1,1)$ $\quad b = (-1,1)$

$f = a \bullet b = 0$



$$a \bullet a = \sum_{i=1}^{d} a_i^2 = \|a\|^2$$

<u>Matrices</u> : tuple of vectors

$P: P_1, P_2 \dots P_m \in \mathbb{R}^n$

$$P = \begin{bmatrix} \leftarrow & P_1 & \rightarrow \\ \leftarrow & P_2 & \rightarrow \\ & \vdots & \\ \leftarrow & P_m & \rightarrow \end{bmatrix}$$

# of rows $\Leftarrow m \times n \Rightarrow$ # of columns

$P \in \mathbb{R}^{m \times n}$

<u>Matrix - vector product</u>

$P \in \mathbb{R}^{m \times n}$ $\quad v \in \mathbb{R}^{n}$

$$a = Pv \qquad a \in \mathbb{R}^m$$

$$a = \left( P_1 \cdot v \ , \ P_2 \cdot v \ \ldots \ P_m \cdot v \right)$$

Example:

$m = 2$
$n = 3$

$$P = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \end{pmatrix} \qquad v = (1, -1, 1)$$

$$Pv = \left( 1\times 1 + 2\times -1 + 0\times 1, \ 0\times 1, \ 1\times -1, \ 1\times 1 \right)$$

$$= (2, 0)$$