CS221 Problem Workout Solutions

Week 9

1) [CA session] Problem 1

Compute the conjunctive normal form (CNF) of the following two formulas and write every step of your computation:

(a) $\neg P \to \neg\neg(Q \vee (R \wedge \neg S))$

(b) $(P \to (Q \vee (R \wedge S))) \wedge (R \vee (S \to Q))$

Solution

(a)

$\neg P \to \neg\neg(Q \vee (R \wedge \neg S))$ Given

$\neg P \to (Q \vee (R \wedge \neg S))$ Double negation

$\neg\neg P \vee (Q \vee (R \wedge \neg S))$ Implication

$P \vee (Q \vee (R \wedge \neg S))$ Double negation

$(P \vee Q \vee R) \wedge (P \vee Q \vee \neg S)$ Distributivity

(b)

$(P \to (Q \vee (R \wedge S))) \wedge (R \vee (S \to Q))$ Given

$(P \to (Q \vee (R \wedge S))) \wedge (R \vee (\neg S \vee Q))$ Implication

$(\neg P \vee (Q \vee (R \wedge S))) \wedge (R \vee (\neg S \vee Q))$ Implication

$(\neg P \vee Q \vee R) \wedge (\neg P \vee Q \vee S) \wedge (R \vee \neg S \vee Q)$ Distributivity.

2) **[CA session] Problem 2: Proof by Resolution**

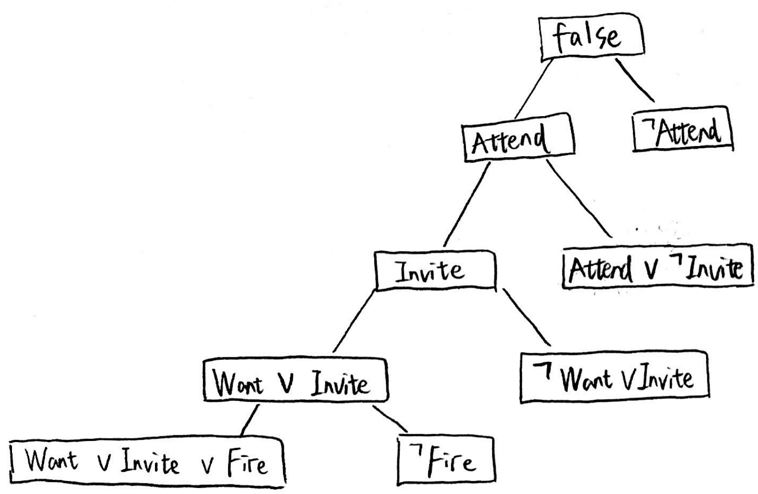
In this question we practice proving by resolution on the following knowledge base: Either Heather attended the meeting or Heather was not invited. If the boss wanted Heather at the meeting, then she was invited. Heather did not attend the meeting. If the boss did not want Heather there, and the boss did not invite her there, then she is going to be fired. Prove Heather is going to be fired.

Solution

$$KB = \{ Attend \lor \neg Invite,\ \neg Want \lor Invite,\ \neg Attend,\ Want \lor Invite \lor Fire \}$$

$$KB' = KB + \text{negation of conclusion}$$

$$= \{ Attend \lor \neg Invite,\ \neg Want \lor Invite,\ \neg Attend,\ Want \lor Invite \lor Fire,\ \neg Fire \}$$



3) [CA Session] Problem 3

Translate the following English sentences into first-order logic formulas:

(a) Every student takes at least one course.

Solution $\forall x\ (\text{Student}(x) \rightarrow \exists y\ (\text{Course}(y) \wedge \text{Takes}(x, y)))$

(b) Every student who takes Analysis also takes Geometry.

Solution $\forall x\ (\text{Student}(x) \wedge \text{Takes}(x, \text{Analysis}) \rightarrow \text{Takes}(x, \text{Geometry})$

(c) No student failed Chemistry but at least one student failed History.

Solution

$\neg(\exists s\ (\text{Student(s)} \wedge \text{Failed}(s, \text{Chemistry}))) \wedge\ (\exists x(\text{Student}(x) \wedge \text{Failed}(x, \text{History})))$

4) Problem 4

Translate each of the following sentences into first order logic using only the predicates listed below:

- Teacher(x): x is a teacher.
- Student(x): x is a student.
- Test(x): x is a test.
- Passed(x, y): x passed y.

(i) [2 point] Some students are also teachers.

Solution $\exists x\ (\text{Student}(x) \wedge \text{Teacher}(x))$

(ii) [3 points] All students have failed a test.

Solution $\forall x\ (\text{Student}(x) \rightarrow \exists y\ (\text{Test(y)} \wedge \neg\ \text{Passed}(x,\ y)))$

(iii) [3 points] There is a test that every student has passed.

Solution $\exists x\ (\text{Test}(x) \wedge \forall y\ (\text{Student}(y) \rightarrow \text{Pass}(y,\ x)))$

5) **Problem 5**

Imagine we are building a knowledge base of propositions in first order logic and want to make inferences based on what we know. We will deal with a simple setting, where we only have three objects in the world: Alice, Carol, and Bob. Our predicates are as follows:

- Employee(x): x is an employee.
- Boss(x): x is a boss.
- Works(x): x works.
- Paid(x): x gets paid.

The knowledge base we have constructed consists of the following propositions:

(a) Boss(Carol)

(b) Employee(Bob)

(c) Paid(Carol) $\wedge$ Works(Carol)

(d) Paid(Alice)

(e) $\forall x$ (Employee(x) $\leftrightarrow \neg$ Boss(x))

(f) $\forall x$ (Employee(x) $\rightarrow$ Works(x))

(g) $\forall x$ ((Paid(x) $\wedge \neg$ Works(x)) $\rightarrow$ Boss(x))

(i) [2 Point] We know from class that one technique we can use to perform inference with our knowledge base is to propositionalize the statements of first-order logic into statements of propositional logic. Practice this by propositionalizing statement (f) from our knowledge base.

Solution (EmployeeAlice $\rightarrow$ WorksAlice) $\wedge$ (EmployeeBob $\rightarrow$ WorksBob) $\wedge$ (EmployeeCarol $\rightarrow$ WorksCarol)

(ii) [3 Points] If we translated the statement "Anyone who is not a boss either works or does not get paid" into first-order logic and added it to our knowledge base, how would the size of the set of valid models representing our knowledge base change, and why?

Solution The set of valid would stay the same as the statement is entailed by our current knowledge base.

(iii) [7 Points] Using only our original knowledge base (not including the statement from part (ii)), we want to answer the question "Does everyone work?" We first translate the sentence "everyone works" into first order logic as statement f. Determine the answer to our query by considering the following questions of satisfiability:

① [3 points] Is KB $\cup \neg f$ satisfiable? Answer yes/no. If yes, fill in the following table with T for true and F for false to show that there is a satisfying model.

x	Employee(x)	Boss(x)	Works(x)	Paid(x)
Alice				
Bob				
Carol				

Solution Yes

x	Employee(x)	Boss(x)	Works(x)	Paid(x)
Alice	F	T	F	T
Bob	T	F	T	T or F
Carol	F	T	T	T

② [3 points] Is KB $\cup f$ satisfiable? Answer yes/no. If yes, fill in the following table with T for true and F for false to show that there is a satisfying model.

x	Employee(x)	Boss(x)	Works(x)	Paid(x)
Alice				
Bob				
Carol				

Solution Yes

x	Employee(x)	Boss(x)	Works(x)	Paid(x)
Alice	T or F	Opposite	T	T
Bob	T	F	T	T or F
Carol	F	T	T	T

③ [1 points] Based on your answers to the previous two parts, does our knowledge base entail f, contradict f, or is f contingent? And what should the answer to our original question "Does everyone work?" be?

Solution f is contingent. Answer should be "maybe" or "it depends"