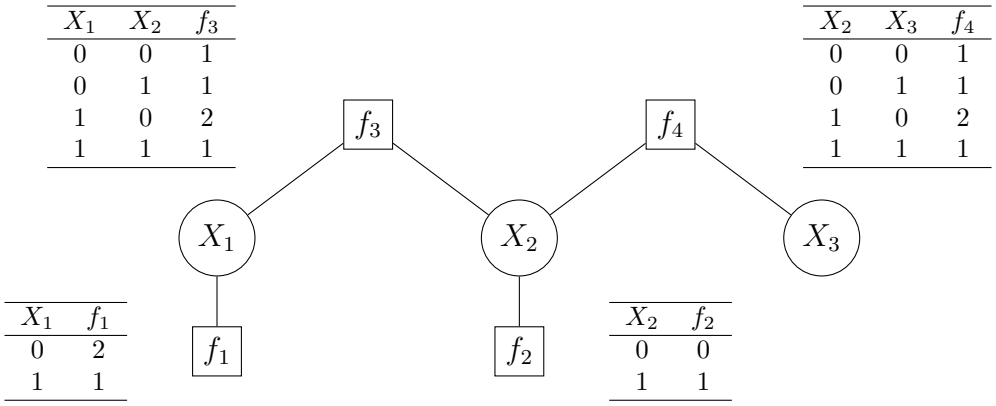
CS221 Problem Session

Week 7

1) Problem 1: Markov Networks

This problem will give you some practice on computing probabilities given a Markov network. Specifically, given the Markov network below, we will ask you questions about the probability distribution $\mathbb{P}(X_1, X_2, X_3)$ over the **binary** random variables $X_1, X_2,$ and X_3.



(a) What is the normalization constant Z (i.e. the total of all possible weights)?

(b) What is $\mathbb{P}(X_1 = 0, X_2 = 0, X_3 = 0)$?

(c) What is $\mathbb{P}(X_1 = 0, X_2 = 1, X_3 = 0)$?

(d) What is $\mathbb{P}(X_2 = 0)$?

(e) What is $\mathbb{P}(X_3 = 0)$?

2) Problem 2: The Bayesian Bag of Candies Model

You have a lot of candy left over from Halloween, and you decide to give them away to your friends. You have four types of candy: **A**pple, **B**anana, **C**aramel, **D**ark-Chocolate. You decide to prepare candy bags using the following process.

- For each candy bag, you first flip a (biased) coin Y which comes up heads ($Y = \text{H}$) with probability λ and tails ($Y = \text{T}$) with probability $1 - \lambda$.
- If Y comes up heads ($Y = \text{H}$), you make a **H**ealthy bag, where you:
  (a) Add one **A**pple candy with probability p_1 or nothing with probability $1 - p_1$;
  (b) Add one **B**anana candy with probability p_1 or nothing with probability $1 - p_1$;
  (c) Add one **C**aramel candy with probability $1 - p_1$ or nothing with probability p_1;
  (d) Add one **D**ark-Chocolate candy with probability $1 - p_1$ or nothing with probability p_1.
- If Y comes up tails ($Y = \text{T}$), you make a **T**asty bag, where you:
  (a) Add one **A**pple candy with probability p_2 or nothing with probability $1 - p_2$;
  (b) Add one **B**anana candy with probability p_2 or nothing with probability $1 - p_2$;
  (c) Add one **C**aramel candy with probability $1 - p_2$ or nothing with probability p_2;
  (d) Add one **D**ark-Chocolate candy with probability $1 - p_2$ or nothing with probability p_2.

For example, if $p_1 = 1$ and $p_2 = 0$, you would deterministically generate: **H**ealthy bags with one **A**pple and one **B**anana; and **T**asty bags with one **C**aramel and one **D**ark-Chocolate. For general values of p_1 and p_2, bags can contain anywhere between 0 and 4 pieces of candy.

Denote A, B, C, D random variables indicating whether or not the bag contains candy of type **A**pple, **B**anana, **C**aramel, and **D**ark-Chocolate, respectively.

(a) Draw the Bayesian network corresponding to process of creating a single bag.

(b) What is the probability of generating a **H**ealthybag containing **A**pple, **B**anana, **C**aramel, and not **D**ark-Chocolate? For compactness, we will use the following notation to denote this possible outcome:

$$(\textbf{H}\text{ealthy}, \{\textbf{A}\text{pple}, \textbf{B}\text{anana}, \textbf{C}\text{aramel}\}).$$

(c) What is the probability of generating a bag containing **A**pple, **B**anana, **C**aramel, and *not* **D**ark-Chocolate?

(d) What is the probability that a bag was a **T**asty one, given that it contains **A**pple, **B**anana, **C**aramel, and *not* **D**ark-Chocolate?