CS221 Problem Session Solutions

Week 9

1) **Problem 1: Conjunctive Normal Form**

Compute the conjunctive normal form (CNF) of the following two formulas and write every step of your computation:

(a) $P \leftrightarrow Q$

Solution

$$\begin{array}{ll} P \leftrightarrow Q & \\ (P \rightarrow Q) \land (Q \rightarrow P) & \text{Definition} \\ (\neg P \lor Q) \land (\neg Q \lor P) & \text{Implication rule} \end{array}$$

(b) $\neg P \rightarrow \neg\neg(Q \lor (R \land \neg S))$

Solution

$$\begin{array}{ll} \neg P \rightarrow \neg\neg(Q \lor (R \land \neg S)) & \\ \neg P \rightarrow (Q \lor (R \land \neg S)) & \text{Remove double negative} \\ \neg\neg P \lor (Q \lor (R \land \neg S)) & \text{Implication rule} \\ P \lor (Q \lor (R \land \neg S)) & \text{Double negative again} \\ (P \lor Q \lor R) \land (P \lor Q \lor \neg S) & \text{Distributing} \end{array}$$

(c) $(P \rightarrow (Q \lor (R \land S))) \land (R \lor (S \rightarrow Q))$

Solution

$$\begin{array}{ll} (P \rightarrow (Q \lor (R \land S))) \land (R \lor (S \rightarrow Q)) & \\ (P \rightarrow (Q \lor (R \land S))) \land (R \lor (\neg S \lor Q)) & \text{Implication rule} \\ (\neg P \lor (Q \lor (R \land S))) \land (R \lor (\neg S \lor Q)) & \text{Implication rule} \\ (\neg P \lor Q \lor R) \land (\neg P \lor Q \lor S) \land (R \lor \neg S \lor Q) & \text{Distributing} \end{array}$$

2) **Problem 2: Proof by Resolution** In this question we practice proving by resolution on the following knowledge base:

Either Heather attended the meeting or Heather was not invited. If the boss wanted Heather at the meeting, then she was invited. Heather did not attend the meeting. If the boss did not want Heather there, and the boss did not invite her there, then she is going to be fired. We are going to (unfortunately) prove Heather is going to be fired.

(a) Write out the knowledge base KB based on the above facts. Convert all formulas to CNF to save ourselves time later. Hint: you should use four propositional symbols.

Solution The second and forth facts requires conversion to CNF, $\text{Want} \rightarrow \text{Invite}$ becomes $\neg\text{Want} \vee \text{Invite}$, and $(\neg\text{Want} \wedge \neg\text{Invite}) \rightarrow \text{Fire}$ becomes $\text{Want} \vee \text{Invite} \vee \text{Fire}$ by distributing the negation to the first term and replacing the $\rightarrow$ with $\vee$.

$$\text{KB} = \{\text{Attend} \vee \neg\text{Invite},\ \neg\text{Want} \vee \text{Invite},\ \neg\text{Attend},\ \text{Want} \vee \text{Invite} \vee \text{Fire}\}$$

(b) Write down the steps for the resolution-based inference algorithm in the context of this problem.

Solution Want to show that Heather is going to be fired. This means that if we added Fire to KB, that it would be entailment (not new information). Since the opposite of entailment is contradiction, we can equivalently check if adding ¬Fire is contradiction, so that Fire would be entailment. To do that we:

i. Add ¬Fire to KB.
ii. Convert all formulas in the KB to CNF (done!)
iii. Repeatedly apply resolution:

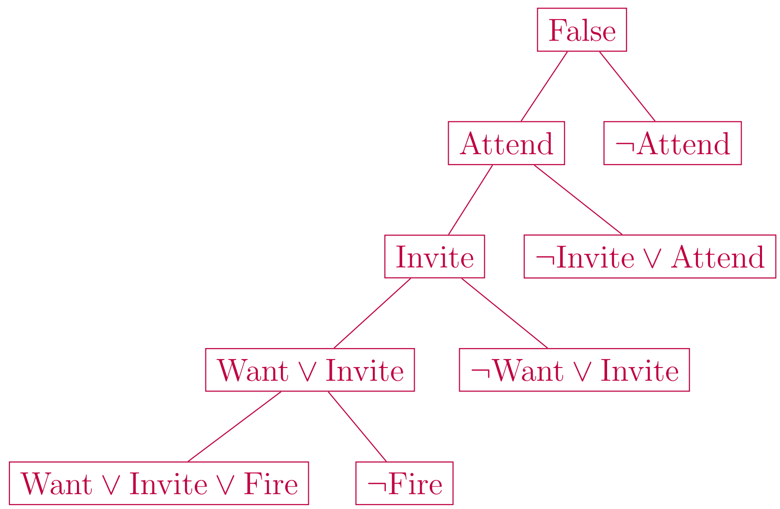
$$\frac{f_1 \vee \cdots \vee f_n \vee p, \quad \neg p \vee g_1 \vee \cdots \vee g_m}{f_1 \vee \cdots \vee f_n \vee g_1 \vee \cdots \vee g_m}$$

Meaning that we find p such that p and $\neg p$ are in KB and continue to derive conclusions.

iv. Return entailment iff we derive false.

(c) Use the resolution algorithm to prove that Heather is going to be fired.

Solution



3) **Problem 3: First Order Logic**

Translate the following sentences into first-order logic formulas (hint: people from Utah are 'Utahns'):

(a) Every person from Utah has visited at least one National Park.

Solution People from Utah are called Utahns, so $\text{Utahn}(x)$ means that x is a person from Utah. For binary formulas we will use $\text{Likes}(x, y)$ and $\text{Visited}(x, y)$.

$$\forall x \, (\text{Utahn}(x) \implies \exists y (\text{National Park}(y) \wedge \text{Visited}(x, y))$$

(b) Every person from Utah who likes fry sauce also likes french fries.

Solution

$$\forall x \, (\text{Utahn}(x) \wedge \text{Likes}(x, \text{Fry Sauce}) \implies \text{Likes}(x, \text{French Fries}))$$

(c) No person from Utah likes Green Jello Salad but at least one person from Utah likes Pioneer Day.

Solution

$$(\neg \exists x \, (\text{Utahn}(x) \wedge \text{Likes}(x, \text{Green Jello Salad}))) \wedge (\exists y \, (\text{Utah}(y) \wedge \text{Likes}(y, \text{Pioneer Day})))$$

(d) Some Utahns live in California.

Solution

$$\exists x (\text{Utahn}(x) \wedge \text{Lives}(x, \text{California}))$$

Or

$$\exists x (\text{Utahn}(x) \wedge \text{LivesInCalifornia}(x))$$

4) **Problem 4: Knowledge Base**

Imagine we are building a knowledge base of propositions in first order logic and want to make inferences based on what we know. We will deal with a simple setting, where we only have three objects in the world: Alice, Carol, and Bob. Our predicates are as follows:

- Employee(x): x is an employee.
- Boss(x): x is a boss.
- Works(x): x works.
- Paid(x): x gets paid.

The knowledge base we have constructed consists of the following propositions:

(a) Boss(Carol)

(b) Employee(Bob)

(c) Paid(Carol) $\wedge$ Works(Carol)

(d) Paid(Alice)

(e) $\forall x$ (Employee(x) $\leftrightarrow \neg$ Boss(x))

(f) $\forall x$ (Employee(x) $\rightarrow$ Works(x))

(g) $\forall x$ ((Paid(x) $\wedge \neg$ Works(x)) $\rightarrow$ Boss(x))

(a) We know from class that one technique we can use to perform inference with our knowledge base is to propositionalize the statements of first-order logic into statements of propositional logic. Practice this by propositionalizing statement (6) from our knowledge base.

Solution (EmployeeAlice $\rightarrow$ WorksAlice) $\wedge$ (EmployeeBob $\rightarrow$ WorksBob) $\wedge$ (EmployeeCarol $\rightarrow$ WorksCarol)

(b) If we translated the statement "Anyone who is not a boss either works or does not get paid" into first-order logic and added it to our knowledge base, how would the size of the set of valid models representing our knowledge base change, and why?

Solution The set of valid would stay the same as the statement is entailed by our current knowledge base.

(c) Using only our original knowledge base (not including the statement from part (b)), we want to answer the question "Does everyone work?" We first translate the sentence "everyone works" into first order logic as statement f. Determine the answer to our query by considering the following questions of satisfiability:

① Is KB $\cup \neg f$ satisfiable? Answer yes/no. If yes, fill in the following table with T for true and F for false to show that there is a satisfying model.

x	Employee(x)	Boss(x)	Works(x)	Paid(x)
Alice				
Bob				
Carol				

Solution Yes

x	Employee(x)	Boss(x)	Works(x)	Paid(x)
Alice	F	T	F	T
Bob	T	F	T	T or F
Carol	F	T	T	T

② Is KB $\cup$ f satisfiable? Answer yes/no. If yes, fill in the following table with T for true and F for false to show that there is a satisfying model.

x	Employee(x)	Boss(x)	Works(x)	Paid(x)
Alice				
Bob				
Carol				

Solution Yes

x	Employee(x)	Boss(x)	Works(x)	Paid(x)
Alice	T or F	Opposite	T	T
Bob	T	F	T	T or F
Carol	F	T	T	T

③ Based on your answers to the previous two parts, does our knowledge base entail f, contradict f, or is f contingent? And what should the answer to our original question "Does everyone work?" be?

Solution f is contingent. Answer should be "maybe" or "it depends"