CS221 Problem Workout

Week 5

Stanford University

Zero Sum Turn Based Games

- Zero Sum (Adversarial)
  - Only one player can win
  - One player loses by the amount the other player wins
- Turn based
  - Only one player takes an action at a time

Image Credit: Chess.com

Game Tree

- In order to reason about games we make a Game Tree
- Enumerate all the possible actions by a given player on their turn
- Allows us to compute expected value of the game based on players policies

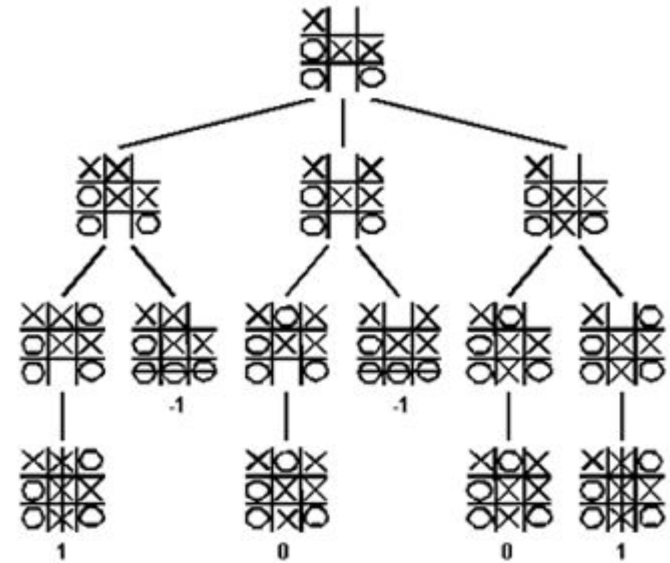
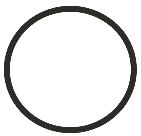
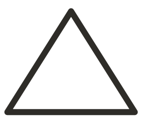
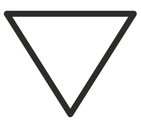
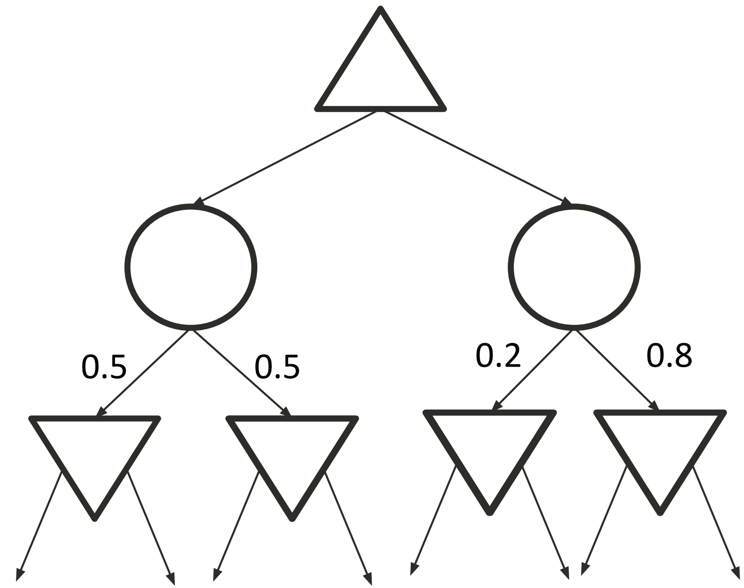


Image Credit: USC

Game Tree

- Helps to represent players based on their policy
-  = Probabilistic
-  = Maximizer
-  = Minimizer
- It is important to consider that a minimizer player is “maximizing” the opponent reward (their reward) in a zero sum game!



Finding Optimal Policy

- We need to evaluate the expected utility of each game state

- Depending on the game we can use:
    - Expectimax: Fixed Random Opponent
    - Minimax: Minimizer Opponent
    - Expectiminimax: Minimizer Opponent with randomness in the game

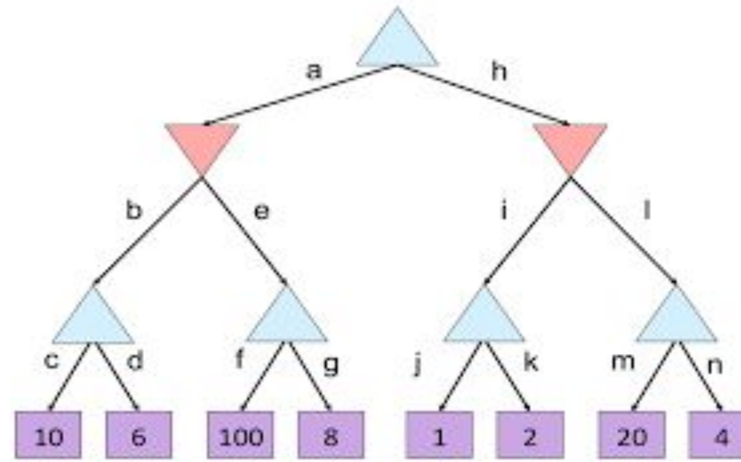


Image Credit: [UC Berkeley]

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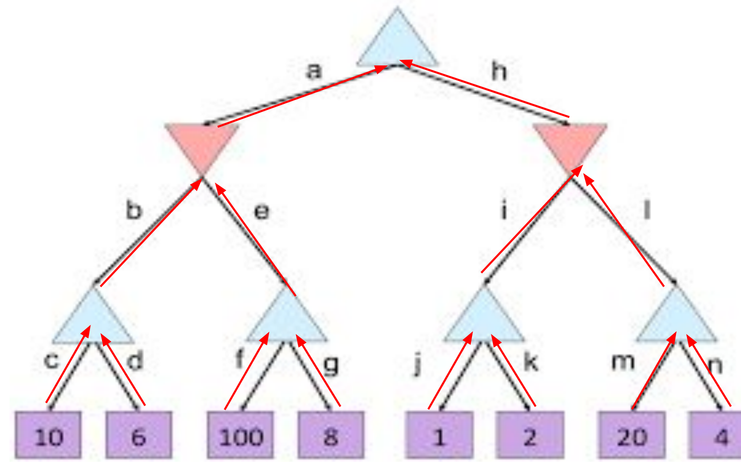


Image Credit: [UC Berkeley]

Improve Efficiency: Evaluation Functions

- Sometimes we can't possibly enumerate the whole search tree
- We can perform a depth limited search to a certain depth in the tree
- We can then define Eval(s) functions which take in a state and return a predicted value of that state

Example: chess

$$\text{Eval}(s) = \text{material} + \text{mobility} + \text{king-safety} + \text{center-control}$$

$$\text{material} = 10^{100}(K - K') + 9(Q - Q') + 5(R - R') + 3(B - B' + N - N') + 1(P - P')$$

$$\text{mobility} = 0.1(\text{num-legal-moves} - \text{num-legal-moves}')$$

...

Improve Efficiency: Alpha Beta Pruning

a_s: lower bound on the value that a max node can contribute upwards (increases with updates)

b_s: upper bound on the value that a min node can contribute upwards (decreases with updates)

alpha_s: maximum a that we know of from currNode to root

beta_s: minimum b that we know of from currNode to root

A max node only has a chance of being on the optimal path if **a_s ≤ beta_s**

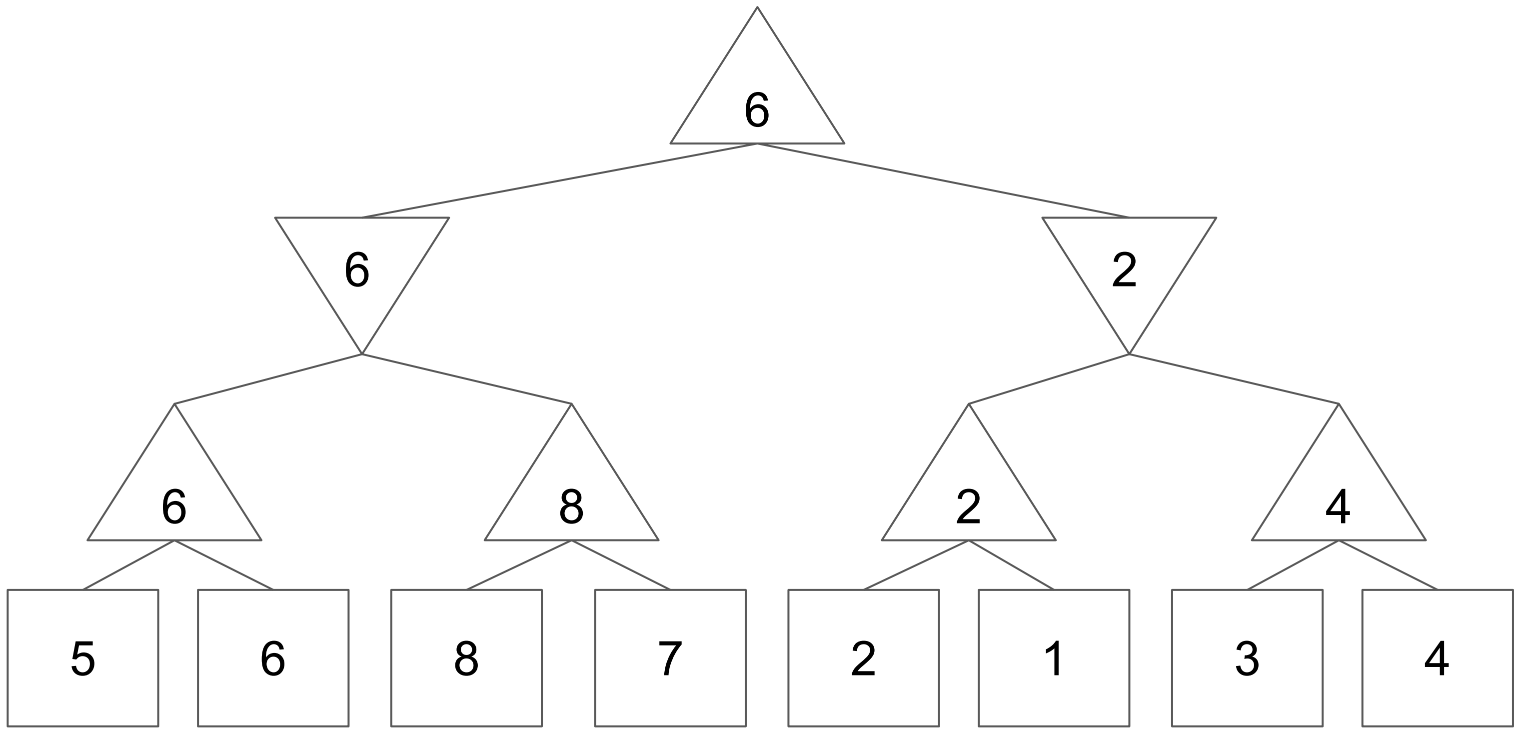
- "My value will be at least a_s, my min ancestors will let through at most beta_s"

If we see a max node where a_s > beta_s: we can prune all of its unexplored children!

- Exploring more children will only increase the max node's value, which is already not feasible through the min ancestors

Work this out for min nodes!

Improve Efficiency: Alpha Beta Pruning



2) "I am the Lorax who speaks for the [game] trees, which you seem to be [alpha-beta pruning] as fast as you please!" - The Lorax

(a) Evaluate the following game (Figure 1) where the edges are probabilities:

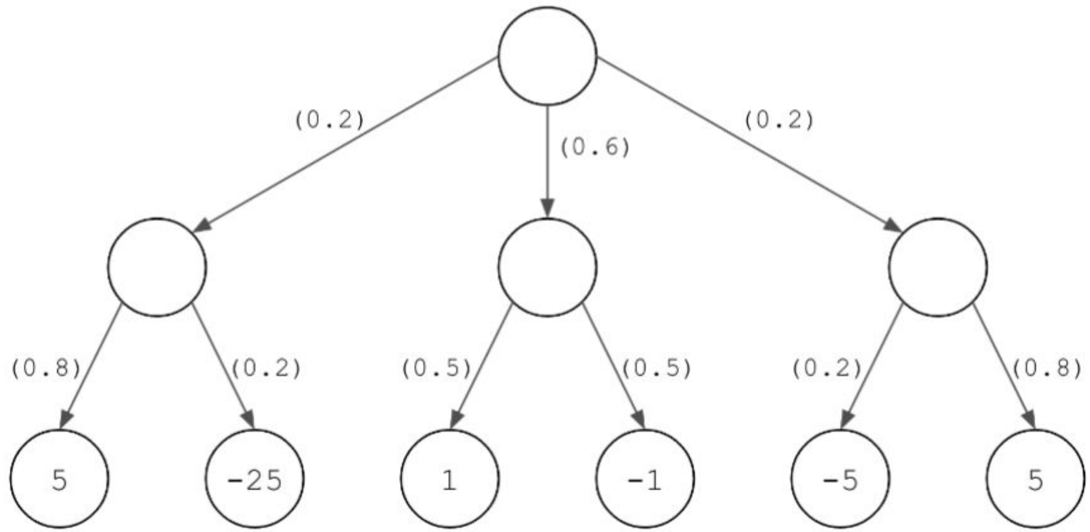


Figure 1

Pretend the top node is now a maximizing player. Under expectimax which action should they take (left, center, or right) and what is the value of the game.

(b) Evaluate the game in Figure 2 using the minimax strategies for both players, with $x = -5$. Recall that upwards pointing triangles is the maximizing player and downwards pointing is the minimizing player.

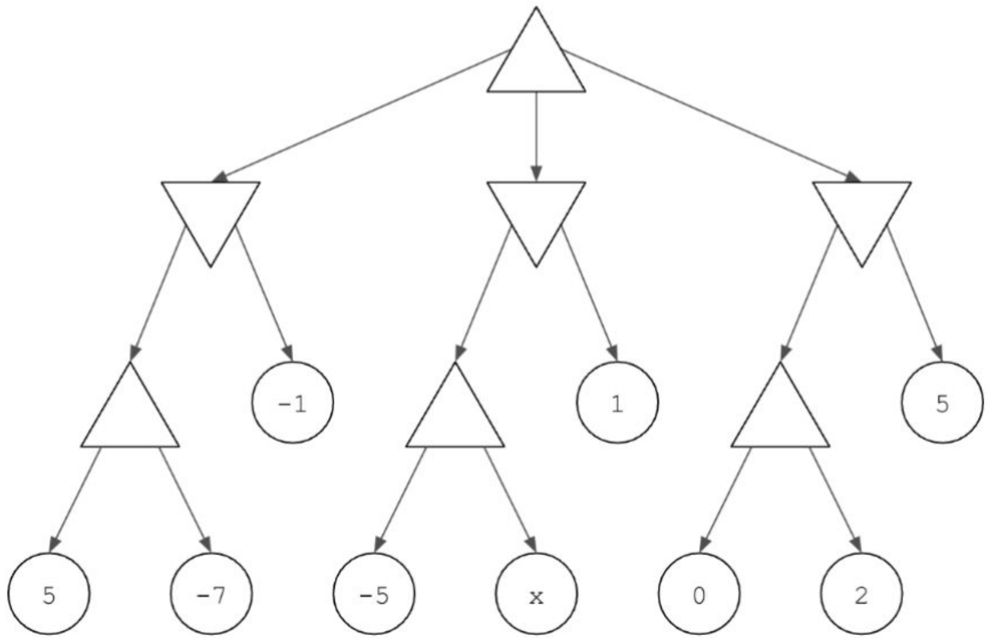


Figure 2

Can we pick x so that the maximizing player loses? Why or why not.

(c) Can either player do better by deviating from minimax assuming the other stays?

(d) Evaluate the game in Figure 3 under the expectiminimax strategy, using $x = -5$. Write down a funny answer for who the third player playing the circles is.

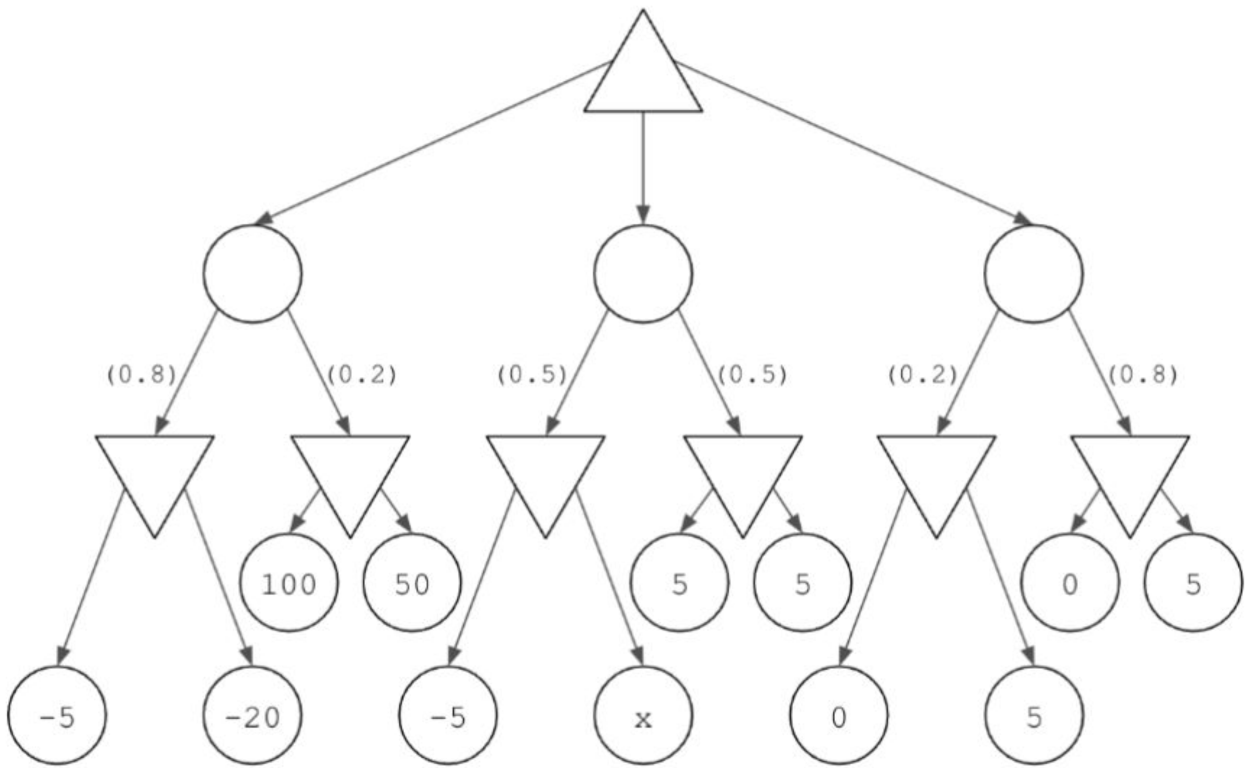


Figure 3

(e) In the previous problem, is there a value of x we can choose so that the game does not end in a draw?

(f) Assume that in the case of a tie in the value of multiple options, the maximizing player chooses the rightmost tied-value action. Still referring to (d) and Figure 3 with $x = -5$, explain, in your own words, why expectiminimax always chooses to draw the game given this choice of tie-breaking. Is there a better way of breaking ties?

Problem 1: General ML Review

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“Linear Regression” with Feature Maps

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We have a trained linear regression model $f_{\mathbf{w}}(x) = \mathbf{w} \cdot \phi(x)$. In your own words, explain why we call this model “linear”. Is it linear in x? Linear in $\phi(x)$? Linear in $\mathbf{w}$? Note that linearity for some generic function g means that $g(x + y) = g(x) + g(y)$ and $g(\alpha x) = \alpha g(x)$ for all parameters α.

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- Is it linear in x? NO!
- Linear in $\phi(x)$? Yes
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Key Takeaway: Feature maps let us express / model non-linear functions within linear regression!

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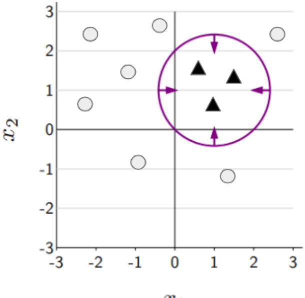
Key Takeaway: Feature maps let us express non-linear functions within linear classification models, e.g. quadratic features:



Problem 1: General ML Review

Linear Classification Decision Boundaries

Linear Classification Decision Boundaries

We are working with a classification model $f_{\mathbf{w}}(x) = \text{sign}(\mathbf{w} \cdot \phi(x))$. What is the decision boundary? What does $\mathbf{w} \cdot \phi(x)y = -1000$ imply about how well our model classified the point (x, y)? What does $\mathbf{w} \cdot \phi(x)y = 0.1$ imply about how well our model classified the point (x, y)?

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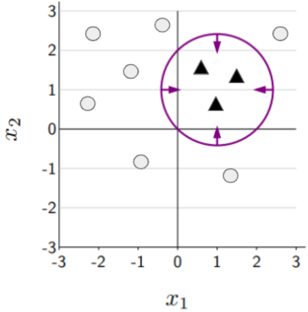

Recall our definition: The decision boundary is $w \cdot \phi(x) = 0$.
Key Takeaway: Decision boundaries let us separate data into different groups!

Linear Classification Decision Boundaries

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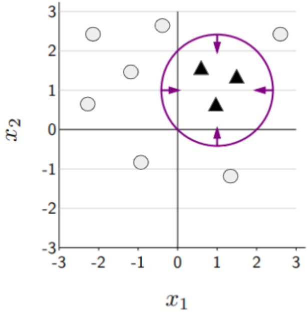

Our model is confident in the classification (far from the decision boundary), but incorrect in the classification (note the sign).

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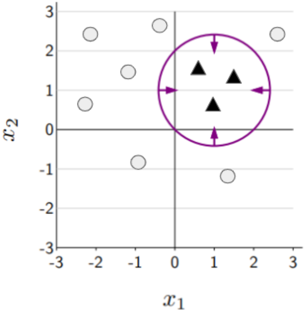

Our model is not very confident in the classification (close to the decision boundary), but correct in the classification (same signs).

Problem 1: General ML Review

Loss Functions

Loss Functions

Additionally, you consider using the following loss function

$$1[(\mathbf{w} \cdot \phi(x))y \leq 0]$$

for gradient descent. Explain why using this loss function is a bad idea.

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This is the zero-one loss function, which has zero gradient almost everywhere!

Key Takeaway: We want our loss function to have a meaningful gradient for gradient descent!

Loss Functions

After solving the prior problem, you realize the zero-one loss function is a bad idea and instead decide to use the logistic loss function. Your data is $y \in \{0, +1\}$, so you define the logistic loss as follows

$$L(x, y; \mathbf{w}) = -y \log(f(x; \mathbf{w})) - (1 - y) \log(1 - f(x; \mathbf{w}))$$

where f has a range of $[0, 1]$. Before picking f, you'd like to differentiate L with respect to $\mathbf{w}$. Is this possible, and if so, what is $\frac{\partial L}{\partial \mathbf{w}}$?

Loss Functions

$$L(x, y; \mathbf{w}) = -y \log(f(x; \mathbf{w})) - (1 - y) \log(1 - f(x; \mathbf{w}))$$

Yes! We use the chain rule:

$$\begin{aligned}\frac{\partial L(x, y; \mathbf{w})}{\partial \mathbf{w}} &= -y \frac{1}{f(x; \mathbf{w})} \frac{\partial f(x; \mathbf{w})}{\partial \mathbf{w}} + (1 - y) \frac{1}{1 - f(x; \mathbf{w})} \frac{\partial f(x; \mathbf{w})}{\partial \mathbf{w}} \\ &= \left(\frac{f(x; \mathbf{w}) - y}{f(x; \mathbf{w})(1 - f(x; \mathbf{w}))} \right) \frac{\partial f(x; \mathbf{w})}{\partial \mathbf{w}}\end{aligned}$$

Key Takeaway: Be prepared to take derivatives of any loss function!

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(Food for thought: how would the derivative change if it were over a summation?)

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(Food for thought: how would the derivative change if it were over a summation?)

Same process, just with indexing!

Loss Functions

For your function f in the above loss function, you can't decide between using the sigmoid function,

$$g(x; \mathbf{w}) = \frac{1}{1 + e^{-\mathbf{w}^T x}}$$

or the shifted tanh function,

$$h(x; \mathbf{w}) = \frac{1}{2}\tanh(\mathbf{w}^T x) + \frac{1}{2} \quad \text{with} \quad \tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

in place of f. How would the derivative from Part (c) look like with function g above in place of f, and with function h above in place of f?

Loss Functions

Sigmoid function,

$$g(x; \mathbf{w}) = \frac{1}{1 + e^{-\mathbf{w}^T x}}$$

$$\frac{\partial g(x; \mathbf{w})}{\partial \mathbf{w}} =$$

Loss Functions

Sigmoid function,

$$g(x; \mathbf{w}) = \frac{1}{1 + e^{-\mathbf{w}^T x}}$$

$$\begin{aligned}
\frac{\partial g(x; \mathbf{w})}{\partial \mathbf{w}} &= -(1 + e^{-\mathbf{w}^T x})^{-2} \frac{\partial}{\partial \mathbf{w}} \left(1 + e^{-\mathbf{w}^T x}\right) \\
&= \frac{x e^{-\mathbf{w}^T x}}{(1 + e^{-\mathbf{w}^T x})^2} \quad \text{(this is a valid answer)} \\
&= x \frac{1}{(1 + e^{-\mathbf{w}^T x})} \frac{e^{-\mathbf{w}^T x}}{(1 + e^{-\mathbf{w}^T x})} \\
&= x g(x; \mathbf{w})(1 - g(x; \mathbf{w}))
\end{aligned}$$

Remember: $\sigma(\mathbf{w})(1 - \sigma(\mathbf{w}))\frac{\partial \mathbf{w}}{\partial x}$ form to save time and work!

Loss Functions

$$L(x, y; \mathbf{w}) = -y \log(f(x; \mathbf{w})) - (1 - y) \log(1 - f(x; \mathbf{w}))$$

$$\frac{\partial L(x, y; \mathbf{w})}{\partial \mathbf{w}} = \left(\frac{f(x; \mathbf{w}) - y}{f(x; \mathbf{w})(1 - f(x; \mathbf{w}))} \right) \frac{\partial f(x; \mathbf{w})}{\partial \mathbf{w}}$$

For sigmoid g:

$$\begin{aligned} \frac{\partial L(x, y; \mathbf{w})}{\partial \mathbf{w}} &= \left(\frac{g(x; \mathbf{w}) - y}{g(x; \mathbf{w})(1 - g(x; \mathbf{w}))} \right) \frac{\partial g(x; \mathbf{w})}{\partial \mathbf{w}} \\ &= \left(\frac{g(x; \mathbf{w}) - y}{g(x; \mathbf{w})(1 - g(x; \mathbf{w}))} \right) g(x; \mathbf{w})(1 - g(x; \mathbf{w}))x \\ &= x(g(x; \mathbf{w}) - y) \end{aligned}$$

Loss Functions

tanh function,

$$h(x;\mathbf{w}) = \frac{1}{2}\tanh(\mathbf{w}^T x) + \frac{1}{2} \quad \text{with} \quad \tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$\begin{aligned}
\frac{\partial h(x;\mathbf{w})}{\partial \mathbf{w}} &= \frac{1}{2}\frac{\partial \tanh(\mathbf{w}^T x)}{\partial \mathbf{w}} \\
&= \frac{1}{2}\frac{\partial}{\partial w}\frac{(e^{\mathbf{w}^T x} - e^{-\mathbf{w}^T x})}{(e^{\mathbf{w}^T x} + e^{-\mathbf{w}^T x})} \\
&= \frac{1}{2}\left[\frac{(e^{\mathbf{w}^T x} + e^{-\mathbf{w}^T x})}{(e^{\mathbf{w}^T x} + e^{-\mathbf{w}^T x})} - \frac{(e^{\mathbf{w}^T x} + e^{-\mathbf{w}^T x})^2}{(e^{\mathbf{w}^T x} + e^{-\mathbf{w}^T x})^2}\right] x \\
&= \frac{1}{2}(1 - \tanh(\mathbf{w}^T x)^2)x
\end{aligned}$$

Loss Functions

$$L(x, y; \mathbf{w}) = -y \log(f(x; \mathbf{w})) - (1 - y) \log(1 - f(x; \mathbf{w}))$$

$$\frac{\partial L(x, y; \mathbf{w})}{\partial \mathbf{w}} = \left(\frac{f(x; \mathbf{w}) - y}{f(x; \mathbf{w})(1 - f(x; \mathbf{w}))} \right) \frac{\partial f(x; \mathbf{w})}{\partial \mathbf{w}}$$

For tanh h:

$$\frac{\partial L(x, y; \mathbf{w})}{\partial \mathbf{w}} = \left(\frac{h(x; \mathbf{w}) - y}{h(x; \mathbf{w})(1 - h(x; \mathbf{w}))} \right) \frac{\partial h(x; \mathbf{w})}{\partial \mathbf{w}}$$

$$= \left(\frac{h(x; \mathbf{w}) - y}{\frac{1}{2}(\tanh(\mathbf{w}^T x) + 1)(1 - \frac{1}{2}(\tanh(\mathbf{w}^T x) + 1))} \right) \frac{1}{2}(1 - \tanh(\mathbf{w}^T x)^2) x$$

$$= \left(\frac{h(x; \mathbf{w}) - y}{(\tanh(\mathbf{w}^T x) + 1)\frac{1}{2}(1 - \tanh(\mathbf{w}^T x))} \right) (1 - \tanh(\mathbf{w}^T x)^2) x$$

$$= 2x(h(x; \mathbf{w}) - y)$$

Problem 1: General ML Review

Backpropagation

Backpropagation

Explain why writing the derivative of the loss function in the form of $cx(f(x;\mathbf{w}) - y)$ is very convenient for backpropagation.

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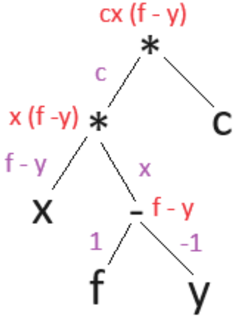

Very straightforward arithmetic operations involving known values!

Key Takeaway: Backpropagation breaks down derivatives into a simple structure for a computer to do!

Problem 1: General ML Review

Reusing Derivatives

Reusing Derivatives

Unfortunately your model has poor performance for both sigmoid and tanh. You decide to make your model a neural network to hopefully fix that.

Let

$$N(x; A, B) = B \max\{Ax, 0\} = z$$

The loss function is now:

$$\begin{aligned} L(x, y; A, B, \mathbf{w}) = -y \log(f(N(x; A, B); \mathbf{w})) - \\ (1 - y) \log(1 - f(N(x; A, B); \mathbf{w})) \end{aligned}$$

Can we we reuse our result from before for $\frac{\partial L}{\partial w}$?

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Can we we reuse our result from before for $\frac{\partial L}{\partial w}$?

Replace x with $z = N(x; A, B)$!

We were differentiating with respect to $\mathbf{w}$, not x, so the process doesn't change! N is simply a constant in this context.

Key Takeaway: Be careful of what you're differentiating with respect to!

Problem 1: General ML Review

Regularization

Regularization

Food for thought: suppose we figure that our model's poor performance was due to overfitting instead. Why might L_2 regularization help, and how would it change our loss function?

$$L(x, y; \mathbf{w}) = -y \log(f(x; \mathbf{w})) - (1 - y) \log(1 - f(x; \mathbf{w}))$$

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Key Takeaway: L_2 regularization penalizes our weights $\mathbf{w}$ when we take a minimization:

$$\min_{\mathbf{w}} \left[L(x, y; \mathbf{w}) = -y \log(f(x; \mathbf{w})) - (1 - y) \log(1 - f(x; \mathbf{w})) + \frac{\lambda}{2} ||\mathbf{w}||_2^2 \right]$$

Search Problem (from Week 3)

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Defining the Search Problem

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In 16th century England, there were a set of $N+1$ cities $C = \{0, 1, 2, \ldots, N\}$. Connecting these cities were a set of bidirectional roads R: $(i, j) \in R$ means that there is a road between city i and city j. Assume there is at most one road between any pair of cities, and that all the cities are connected. If a road exists between i and j, then it takes $T(i, j)$ hours to go from i to j.

Romeo lives in city 0 and wants to travel along the roads to meet Juliet, who lives in city N. They want to meet.

Search problems typically require a lot of reading... try to break it down to the important parts.

Defining the Search Problem

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- $N + 1$ cities $C = \{0, 1, 2, \ldots, N\}$
- R: $(i, j) \in R$ is a road between city i and j
- Only 1 road between any 2 cities
- $T(i, j)$ hours to go along road from i to j
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To reduce confusion, they will reconnect after each traveling a road. For example, if Romeo travels from city 3 to city 5 in 10 hours at the same time that Juliet travels from city 9 to city 7 in 8 hours, then Juliet will wait 2 hours. Once they reconnect, they will both traverse the next road (neither is allowed to remain in the same city). Furthermore, they must meet in the end in a city, not in the middle of a road. Assume it is always possible for them to meet in a city.

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- Romeo starts at 0, Juliet starts at N

- Romeo and Juliet will wait for the other to finish traveling before moving again, i.e.
  Cost of $(r, j) \to (r', j') = \max(T(r, r'), T(j, j'))$

Defining the Search Problem

- $N + 1$ cities $C = \{0, 1, 2, \ldots, N\}$
- R: $(i, j) \in R$ is a road between city i and j
- Only 1 road between any 2 cities
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States: $s = (r, j)$ where $r \in C$ and $j \in C$ are the cities Romeo and Juliet currently in

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$\text{Actions}((r, j)) = \{(r', j') : (r, r') \in R, (j, j') \in R\}$ corresponds to both traveling to a connected city

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$\text{Cost}((r,j),(r',j')) = \max(T(r,r'), T(j,j'))$ is the maximum over the two times

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$\text{Succ}((r, j), (r', j')) = (r', j')$: just the next pair of cities the two end up at

Defining the Search Problem

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IsGoal$((r, j)) = 1[r = j]$ (whether the two are in the same city)

Search Problem (from Week 3)

Redefining for a Heuristic

Redefining for a Heuristic

- $N+1$ cities $C = \{0, 1, 2, \ldots, N\}$
- R: $(i,j) \in R$ is a road between city i and j
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Uniform Cost Search to compute $M(i,k)$, the minimum time it takes one person to travel from city i to city k for all pairs of cities $i, k \in C$.

Give a consistent A* heuristic for the search problem. Your heuristic should take $O(N)$ time to compute, assuming that looking up $M(i,k)$ takes $O(1)$ time.

Redefining for a Heuristic

- $N + 1$ cities $C = \{0, 1, 2, \ldots, N\}$
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- UCS precompute $M(i, k)$, minimum time to go from any city i to any city k; takes $O(1)$ to look up

How to relax the search problem to make use of the additional info? Is there a contradiction anywhere in the criteria?

Redefining for a Heuristic

- $N + 1$ cities $C = \{0, 1, 2, \ldots, N\}$
- R: $(i, j) \in R$ is a road between city i and j
- Only 1 road between any 2 cities
- $T(i, j)$ hours to go along road from i to j
- Romeo starts at 0, Juliet starts at N
- ~~Romeo and Juliet will wait for the other to finish traveling before moving again, i.e. Cost of $(r, j) \to (r', j') = \max(T(r, r'), T(j, j'))$~~
- UCS precompute $M(i, k)$, minimum time to go from any city i to any city k; takes $O(1)$ to look up

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Redefining for a Heuristic

- $N + 1$ cities $C = \{0, 1, 2, \dots, N\}$
- R: $(i, j) \in R$ is a road between city i and j
- Only 1 road between any 2 cities
- $T(i, j)$ hours to go along road from i to j
- Romeo starts at 0, Juliet starts at N
- Romeo and Juliet will only wait for the other to finish traveling if they make it to the goal
- UCS precompute $M(i, k)$, minimum time to go from any city i to any city k; takes $O(1)$ to look up

Key Takeaway: How to relax the search problem to make use of the additional info? Is there a contradiction anywhere in the criteria?

Redefining for a Heuristic

Romeo and Juliet will only wait for the other to finish traveling if they make it to the goal

$$h((r,j)) = \min_{c \in C} \max\{M(r,c), M(j,c)\}.$$

A* heuristic $h(s)$ is consistent if

$$h(s) \leq \text{Cost}(s,a) + h(\text{Succ}(s,a)).$$

so the following needs to be true

$\min_{c \in C} \max\{M(r,c), M(j,c)\} \leq$

$$\text{Cost}((r,j),(r',j')) + \min_{c' \in C} \max\{M(r',c'), M(j',c')\}$$

Redefining for a Heuristic

Romeo and Juliet will only wait for the other to finish traveling if they make it to the goal

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A* heuristic $h(s)$ is consistent if the following is true

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$$\text{Cost}((r,j),(r',j')) + \min_{c' \in C} \max\{M(r',c'), M(j',c')\}$$

The cost on the right-hand side is the original cost function, which has Romeo/Juliet wait at every stop. That makes the right-hand side larger!