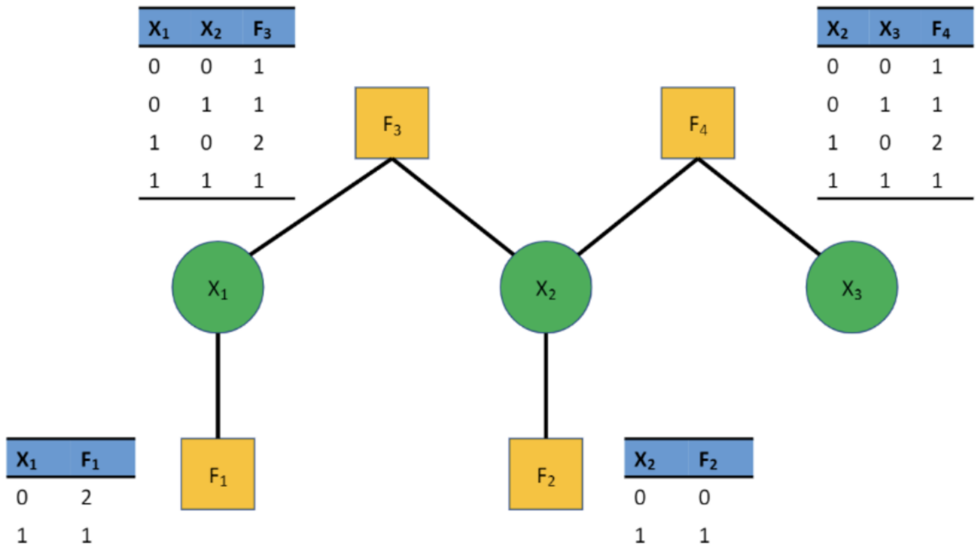
CS221 Problem Workout

Week 7

1) **Problem 1: Markov Networks** This problem will give you some practice on computing probabilities given a Markov network. Specifically, given the Markov network below, we will ask you questions about the probability distribution $p(X_1, X_2, X_3)$ over the binary random variables X_1, X_2, and X_3.



X_1	X_2	F_3
0	0	1
0	1	1
1	0	2
1	1	1

X_2	X_3	F_4
0	0	1
0	1	1
1	0	2
1	1	1

X_1	F_1
0	2
1	1

X_2	F_2
0	0
1	1

(a) What is $p(X_1 = 0, X_2 = 0, X_3 = 0)$?

(b) What is $p(X_1 = 0, X_2 = 1, X_3 = 0)$?

(c) What is $p(X_2 = 0)$?

(d) What is $p(X3 = 0)$?

2) Problem 2: Bayesian Networks Basics

$P(A\|D,X)$			
$+d$	$+x$	$+a$	0.9
$+d$	$+x$	$-a$	0.1
$+d$	$-x$	$+a$	0.8
$+d$	$-x$	$-a$	0.2
$-d$	$+x$	$+a$	0.6
$-d$	$+x$	$-a$	0.4
$-d$	$-x$	$+a$	0.1
$-d$	$-x$	$-a$	0.9

$P(D)$	
$+d$	0.1
$-d$	0.9

$P(X\|D)$		
$+d$	$+x$	0.7
$+d$	$-x$	0.3
$-d$	$+x$	0.8
$-d$	$-x$	0.2

$P(B\|D)$		
$+d$	$+b$	0.7
$+d$	$-b$	0.3
$-d$	$+b$	0.5
$-d$	$-b$	0.5

(a) Given the tables above, draw a minimal representative Bayesian network of this model. Be sure to label all nodes and the directionality of the edges.

(b) Compute the following probabilities: $P(+d \mid +b)$, $P(+d, +a)$, $P(+d \mid +a)$.

(c) Which of the following conditional independences are guaranteed by the above network?

- ☐ $X \perp\!\!\!\perp B \mid D$
  (X and B are cond. ind. given D)
- ☐ $D \perp\!\!\!\perp A \mid X$
- ☐ $D \perp\!\!\!\perp A \mid B$
- ☐ $D \perp\!\!\!\perp X \mid A$

3) Problem 3: Bayesian Networks Trivia

As the president of the National Trivia Association, you must choose between the Bayesians and the Markovians, the nation's top two rival trivia teams, to represent the US at the World Trivia Olympics. To determine the more popular team, you decide to model the change in monthly TV viewership using a Bayesian network.

Let B_t and M_t denote the number of TV viewers that the Bayesians and Markovians have in month t respectively. You have no way of observing these quantities directly, but you can observe two other quantities which they influence: let S_t denote the number of times internet users searched for the Bayesians in month t, and let A_t denote the attendance of the friendly match at a neighborhood pub between the Bayesians and the Markovians in month t.

The viewerships of the two teams evolve according to the following model, where each month a fan is either gained or lost with equal probability:

$$\Pr(M_{t+1}|M_t) = \begin{cases} \frac{1}{2} & \text{if } M_{t+1} = M_t - 1 \\ \frac{1}{2} & \text{if } M_{t+1} = M_t + 1 \\ 0 & \text{otherwise} \end{cases} \qquad \Pr(B_{t+1}|B_t) = \begin{cases} \frac{1}{2} & \text{if } B_{t+1} = B_t - 1 \\ \frac{1}{2} & \text{if } B_{t+1} = B_t + 1 \\ 0 & \text{otherwise} \end{cases}$$

The Bayesian fans like to rewatch their trivia shows by searching the recaps online! We model the fan's size's influence on the number of internet searches by:

$$\Pr(S_t|B_t) = \begin{cases} 0.3 & \text{if } S_t = B_t \\ 0.25 & \text{if } S_t = B_t - 1 \\ 0.2 & \text{if } S_t = B_t - 2 \\ 0.15 & \text{if } S_t = B_t - 3 \\ 0.1 & \text{if } S_t = B_t - 4 \\ 0 & \text{otherwise} \end{cases}$$

Lastly, because most TV viewers attend each monthly friendly matches (although sometimes more, and sometimes fewer), we model the influence of the TV viewership number on the friendly match attendance by:

$$\Pr(A_t|B_t, M_t) = \begin{cases} 0.14 & \text{if } A_t = B_t + M_t \\ 0.13 & \text{if } |A_t - (B_t + M_t)| = 1 \\ 0.11 & \text{if } |A_t - (B_t + M_t)| = 2 \\ 0.09 & \text{if } |A_t - (B_t + M_t)| = 3 \\ 0.06 & \text{if } |A_t - (B_t + M_t)| = 4 \\ 0.04 & \text{if } |A_t - (B_t + M_t)| = 5 \\ 0 & \text{otherwise} \end{cases}$$

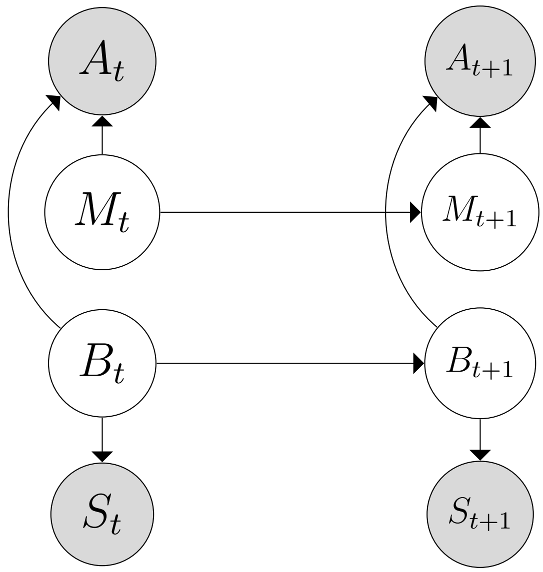


Figure 1: The changing TV viewership count modeled as a dynamic Bayesian network. The unshaded nodes correspond to the latent/hidden TV viewership counts, and the shaded nodes correspond to the observable emissions.

a. (*10 points*) **Inference**

Suppose the Bayesian's trivia team captain took a nationwide poll in month t that concluded they had exactly 75 TV viewers. Suppose additionally that in month $t+2$, the search engine reported 73 people search for the Bayesians online. What is the probability that in month $t+2$ the Bayesians have 77 TV viewers?

$\Pr(B_{t+2} = 77 | B_t = 75, S_{t+2} = 73) =$

b. (*4 points*) **Extra Practice - Gibbs Sampling**

Inference is exhausting; you decide that you'd be satisfied with simply being able to draw samples from distributions rather than specifying them exactly. In particular, you want to sample joint assignments to the variables $\{B_t, M_t, A_t, S_t\}_{t=1}^{T}$ for some time horizon T. You decide to implement Gibbs sampling for this purpose, but something's not right! What additional information, beyond what we've given you, would allow you to perform Gibbs sampling? Briefly explain.

c. (*10 points*) **Particle Filtering**

Throughout this problem, you are free to leave quantities in terms of unevaluated expressions (i.e. you may write $0.75 \cdot 0.5$ instead of 0.375).

Computing all of those terms exactly seems tedious, so you instead decide to employ particle filtering to quickly and painlessly provide you with approximate solutions. You're fine with a (very) crude approximation, so you only use two particles.

(i) [2 points] Suppose you begin with the two particles $(B_1 = 80, M_1 = 75)$ and $(B_1 = 82, M_1 = 74)$. You then observe that $S_1 = 79$ and $A_1 = 154$. Compute the weights that you should assign to the two particles based on this evidence.

(ii) [2 points] Using these weights, we now resample two new particles. Provide this sampling distribution.

Probability of sampling a new particle to be $(B_1 = 80, M_1 = 75) =$

Probability of sampling a new particle to be $(B_1 = 82, M_1 = 74) =$

(iii) [3 points] Suppose both of our new particles are sampled to be $(B_1 = 80, M_1 = 75)$. We now extend these particles using our dynamics models. What is the probability that a particular one of these two particles is extended to:

$(B_1 = 80, M_1 = 75, B_2 = 78, M_2 = 76)$?

$(B_1 = 80, M_1 = 76, B_2 = 79, M_2 = 75)$?