

# CS221 Problem Workout

Oct 14

## 1) [CA session] Problem 1

After finally meeting up, Romeo (R) and Juliet (J) decide to try to catch a goose (G) to keep as a pet. Eventually, they chase it into a  $3 \times 3$  hedge maze show below. Now they play the following turn-based game:

- (a) The Goose moves either Down or Right.
- (b) Romeo moves either Up or Right.
- (c) Juliet moves either Left or Down.

<b>G</b>	<b>o</b>	<b>J</b>
	<b>WALL</b>	
<b>R</b>		

Participants: Goose (G), Romeo (R), Juliet (J), bread (o)

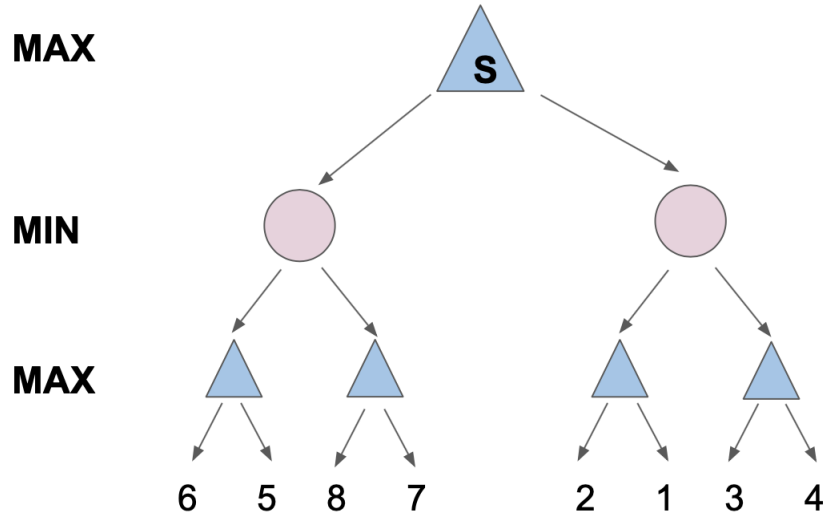
If the Goose enters the square with bread, it gets a reward 1. If either Romeo or Juliet enters the same square as the Goose, they catch it and the Goose gets a reward of  $-50$ . The game ends when either the Goose has been caught or everyone has moved once. Note that it is possible for the Goose to get both rewards.

Construct a depth one minimax tree for the above situation, with the Goose as the maximizer and Juliet and Romeo as the minimizers. Use up-triangles  $\Delta$  for max nodes, down-triangles  $\nabla$  for min nodes, and square nodes for the leaves. Label each node with its minimax value.

What is the minimax value of the game if Romeo defects and becomes a maximizer?

2) [CA session] Problem 2

Consider running alpha-beta pruning on the following minimax tree. Which nodes will be pruned (thus not being visited)?



### 3) [Breakouts] Problem 3

**Consider the speed bump problem we did last week:**

You're programming a self-driving car that can take you from home (position 1) to school (position  $n$ ). At each time step, the car has a current position  $x \in \{1, \dots, n\}$  and a current velocity  $v \in \{0, \dots, m\}$ . The car starts with  $v = 0$ , and at each time step, the car can either increase the velocity by 1, decrease it by 1, or keep it the same; this new velocity is used to advance  $x$  to the new position. The velocity is not allowed to exceed the speed limit  $m$  nor return to 0.

In addition, to prevent people from recklessly cruising down Serra Mall, the university has installed speed bumps at a subset of the  $n$  locations. The speed bumps are located at  $B \subseteq \{1, \dots, n\}$ . The car is not allowed to enter, leave, or pass over a speed bump with velocity more than  $k \in \{1, \dots, m\}$ . **Your goal is to arrive at position  $n$  with velocity 1 in the smallest number of time steps.**

**Now let's add more information to this problem:**

The university wants to remove the old speed bumps and install a single new speed bump at location  $b \in \{1, \dots, n\}$  to maximize the time it takes for the car to go from position 1 to  $n$ .

Let  $T(\pi, B)$  be the time it takes to get from 1 to  $n$  if the car follows policy  $\pi$  if speed bumps  $B$  are present. If  $\pi$  violates the speed limit, define  $T(\pi, B) = \infty$ .

To simplify, assume  $n = 6$  and  $k = 1$ . Again, there is exactly one speed bump. That is,  $B = \{b\}$  with  $b \in \{1, \dots, n\}$ .

$x = 1$ home	$x = 2$	$x = 3$	$x = 4$	$x = 5$	$x = 6$ school
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Figure: The university will add a speed bump somewhere

(i) [5 points] Compute the worst case driving time, assuming you get to adapt your policy to the university's choice of speed bump location  $b$ :  $\max_b \min_{\pi} T(\pi, \{b\})$ . What values of  $b$  attain the maximum?

(ii) [5 points] Compute the best possible time assuming that you have to choose your policy before the university chooses the speed bump:  $\min_{\pi} \max_b T(\pi, \{b\})$ . Make sure to explain your reasoning.