GAMES

CS221: Section 5

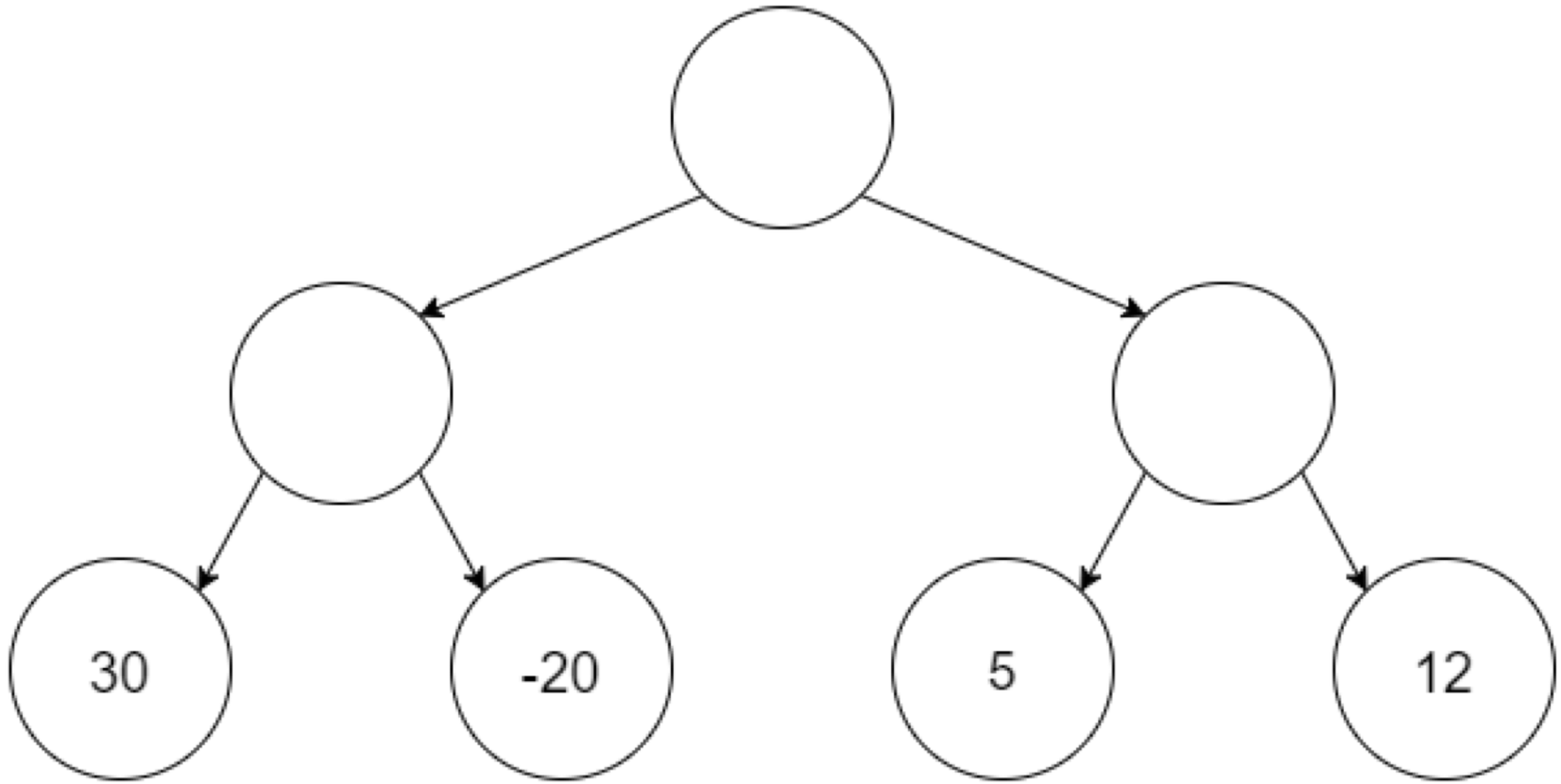
Today's agenda

- Game Trees
- Expectimax
- Minimax

- Game Trees
- Expectimax
- Minimax

Key idea: game tree

Each node is a decision point for a player.
Each root-to-leaf path is a possible outcome of the game.



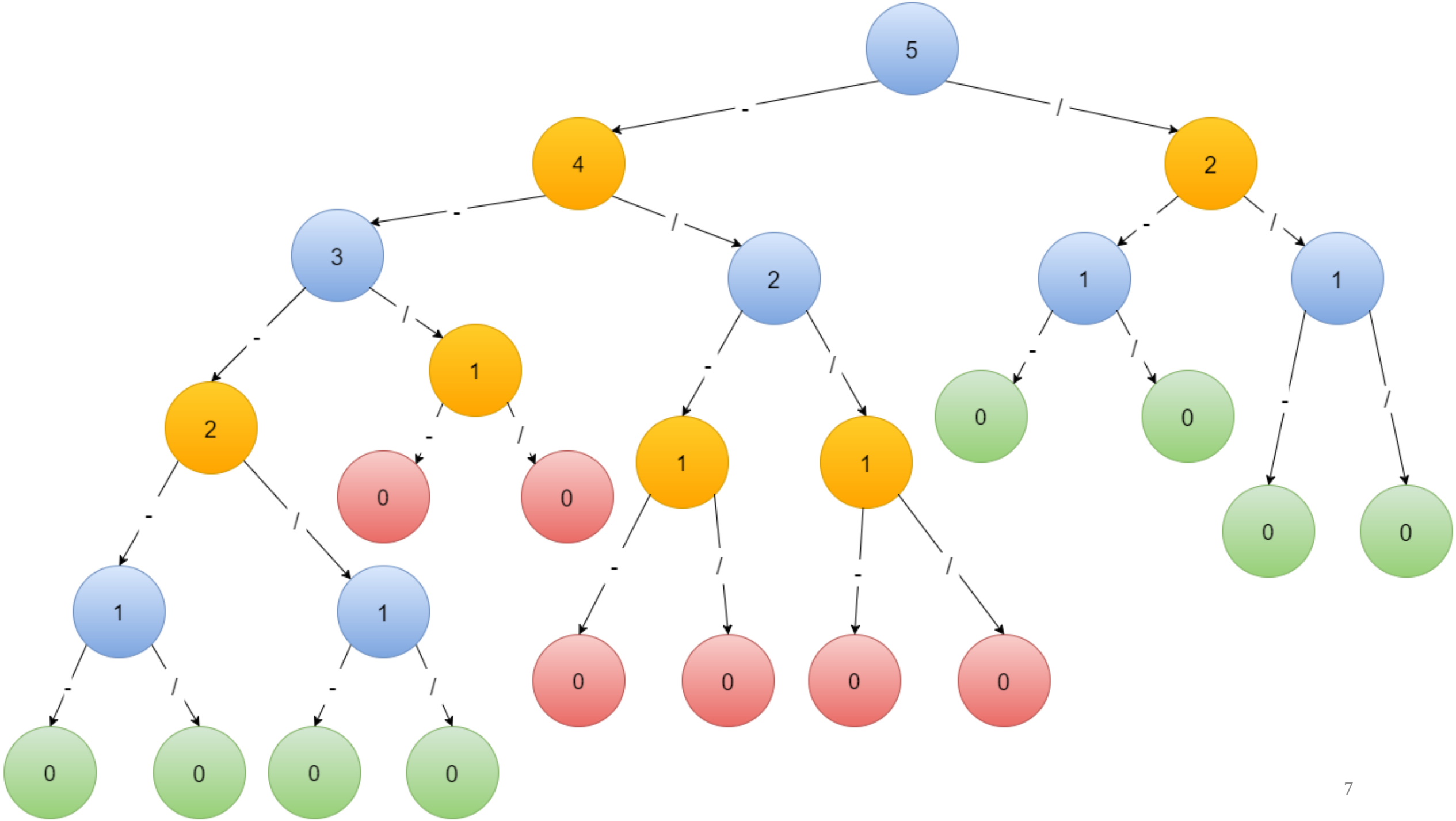
Problem Statement

- Start with a number N
- Players take turns either decrementing N or replacing it with $\left\lfloor \frac{N}{2} \right\rfloor$
- The person to first reach 0 is the winner

Problem Statement

- Start with a number N
- Players take turns either decrementing N or replacing it with $\left\lfloor \frac{N}{2} \right\rfloor$
- The person to first reach 0 is the winner

Lets say N=5


5
4
2
3
2
1
1
2
1
0
0
0
0
1
1
1
1
0
0
0
0
0
0
0
0
0
0
0
0

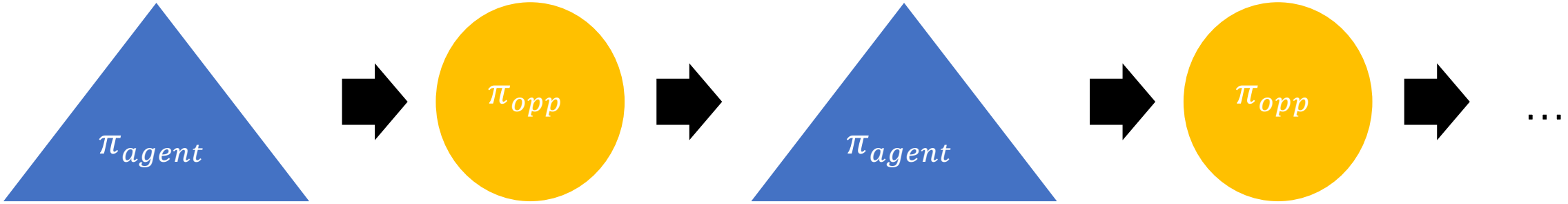
- Game Trees
- Expectimax
- Minimax

Expectimax

The agent chooses the policy that is

optimal with respect to a fixed known policy

Expectimax Recurrence



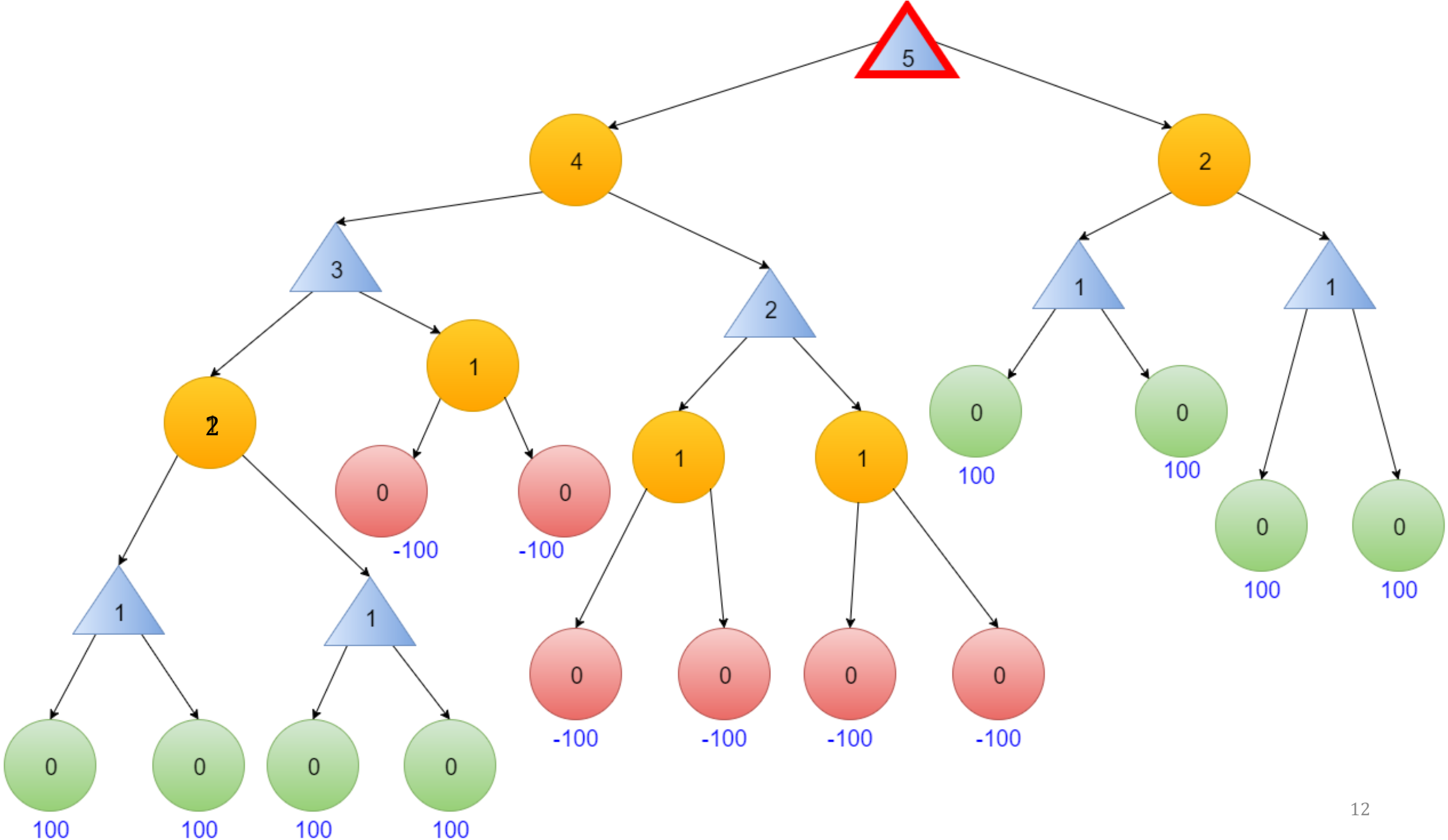
$$V_{opt,\pi}(s) = \begin{cases} \text{Utility}(s) & \text{IsEnd}(s) \\ \max_{a \in \text{Actions}(s)} V_{opt,\pi}\big(Succ(s,a)\big) & \text{Player}(s) = \text{agent} \\ \sum_{a \in \text{Actions}(s)} \pi_{opp}(s,a) V_{opt,\pi}\big(Succ(s,a)\big) & \text{Player}(s) = \text{opp} \end{cases}$$

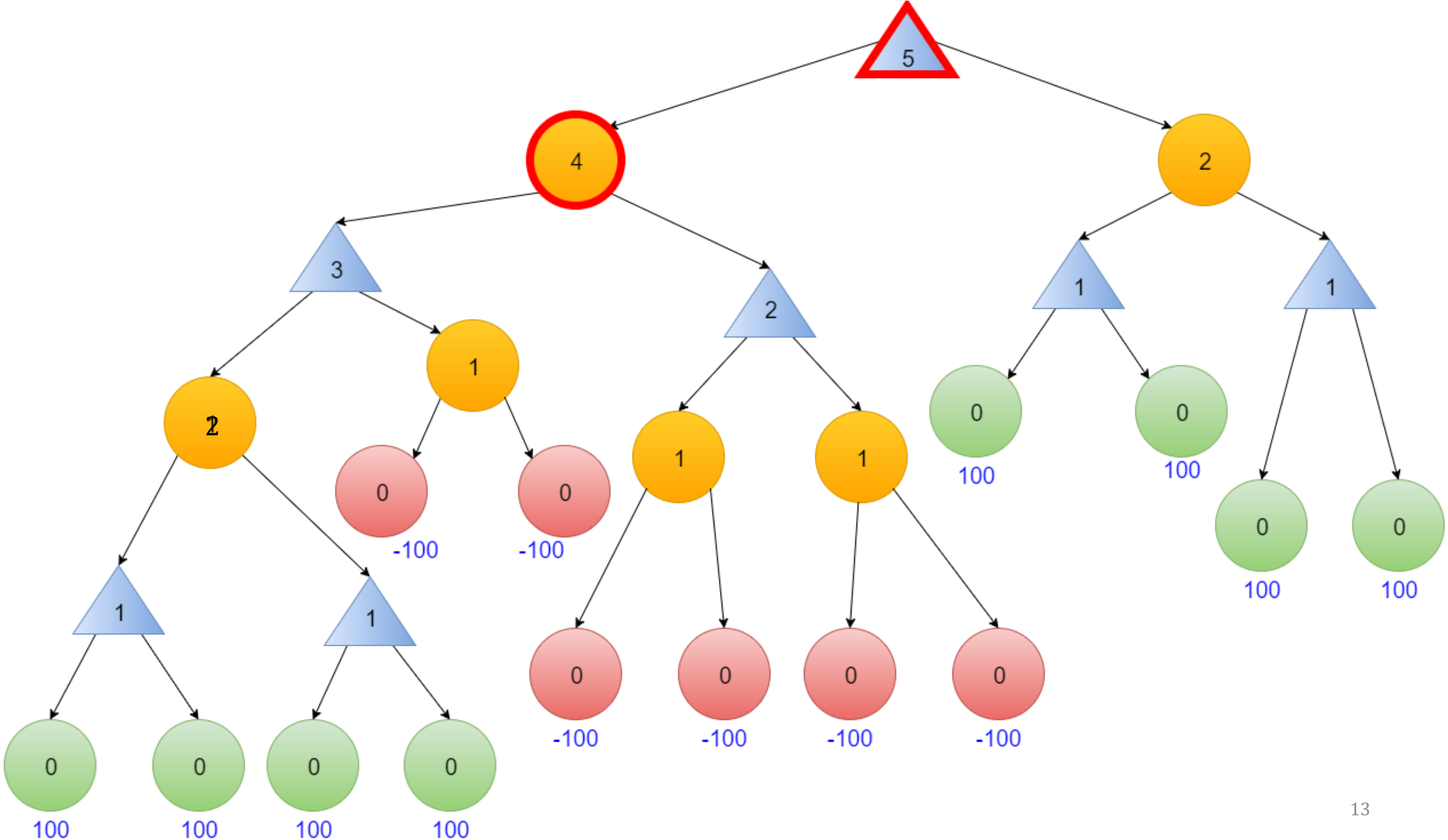
For our example, let's assume that the opponent

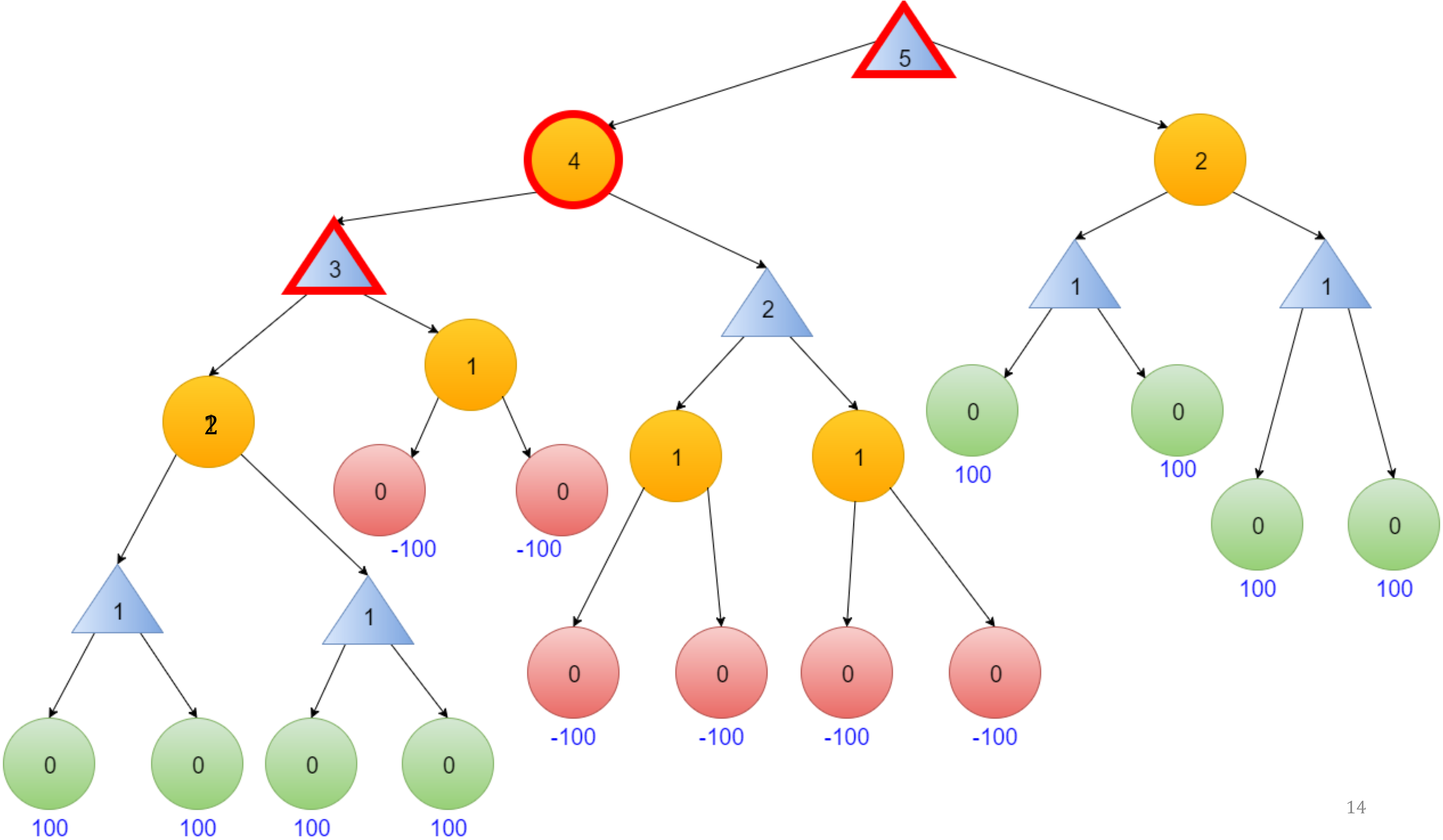
- Decrements if the number is odd
- Uniformly chooses to decrement or halve if the number is even

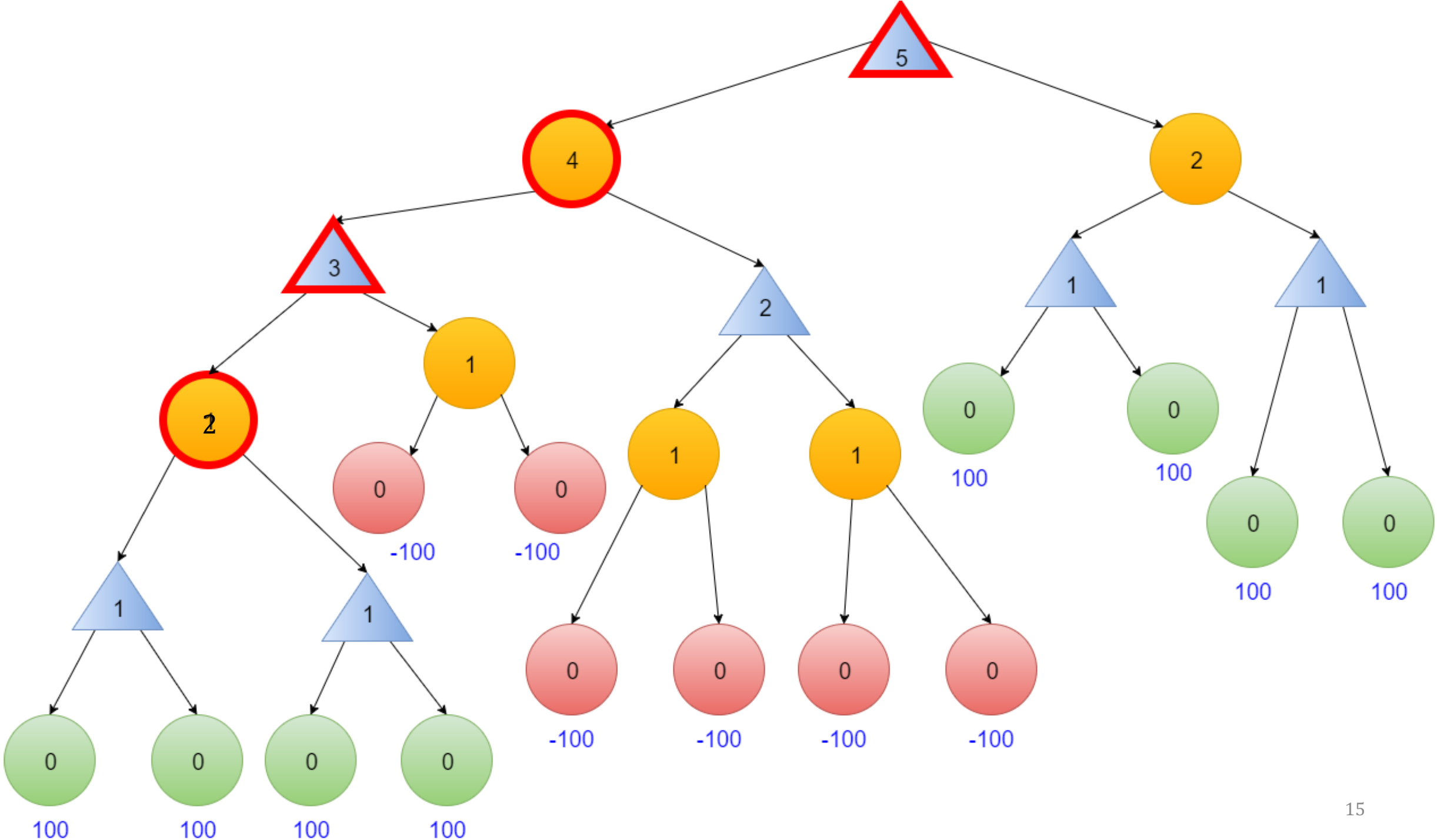
Thus the opponent's policy is given by

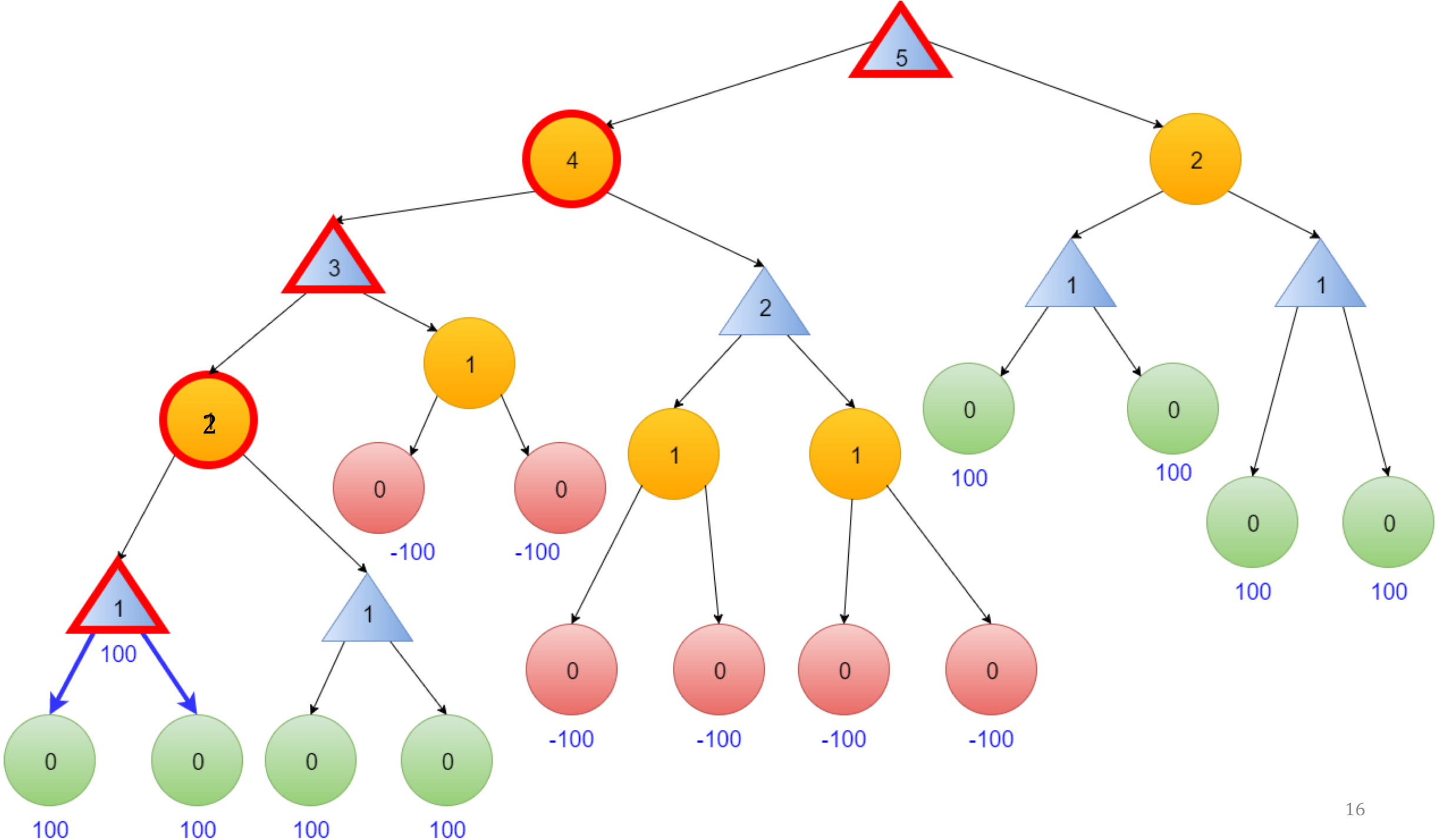
$$\pi_{opp} = \begin{cases} \text{Decrement 1} & \text{Number is odd} \\ \text{Choose uniformly} & \text{Number is even} \end{cases}$$

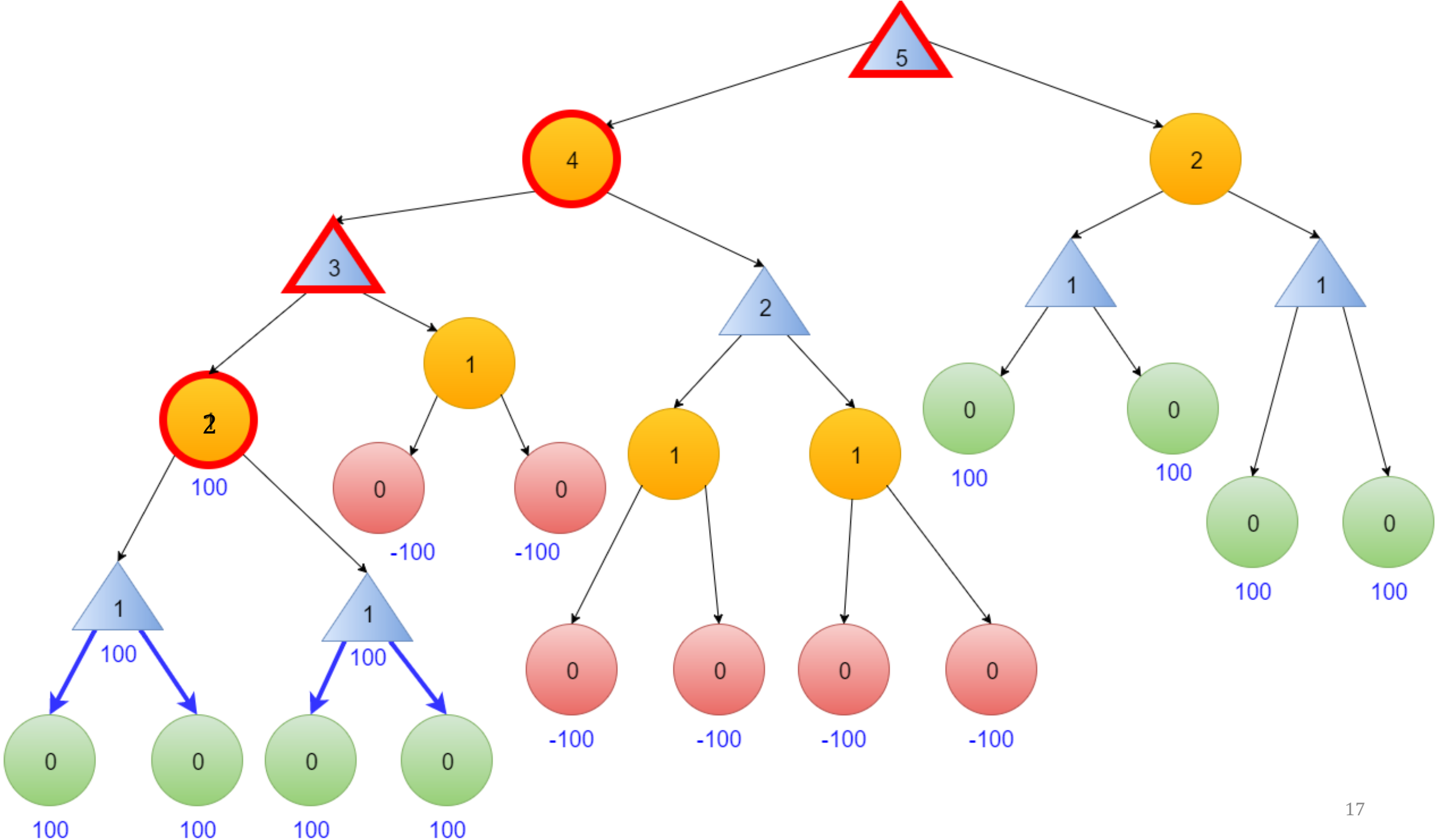

5
4
2
3
2
1
1
1
2
0
0
0
0
0
0
1
1
1
1
0
0
0
0
0
0
0
0
0
0
0
0
0
0
0
0
-100
-100
-100
-100
-100
-100
-100
-100
100
100
100
100
100
100
100
100

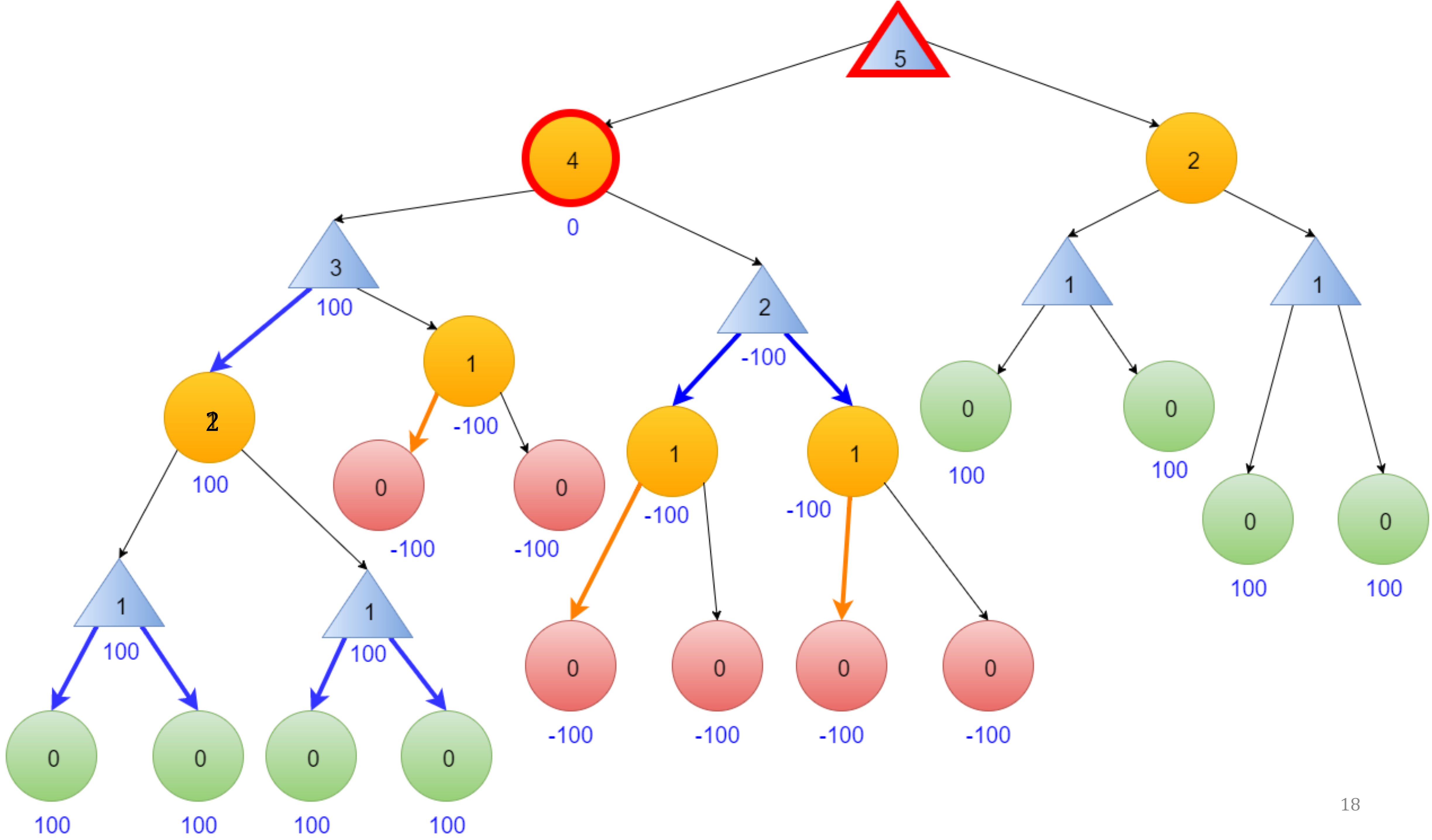

5
4
2
3
1
2
2
1
1
1
0
0
0
0
0
0
0
0
0
0
0
0
0
0
0
0
0
0
0
0
-100
-100
-100
-100
-100
-100
-100
-100
100
100
100
100
100
100
100
100

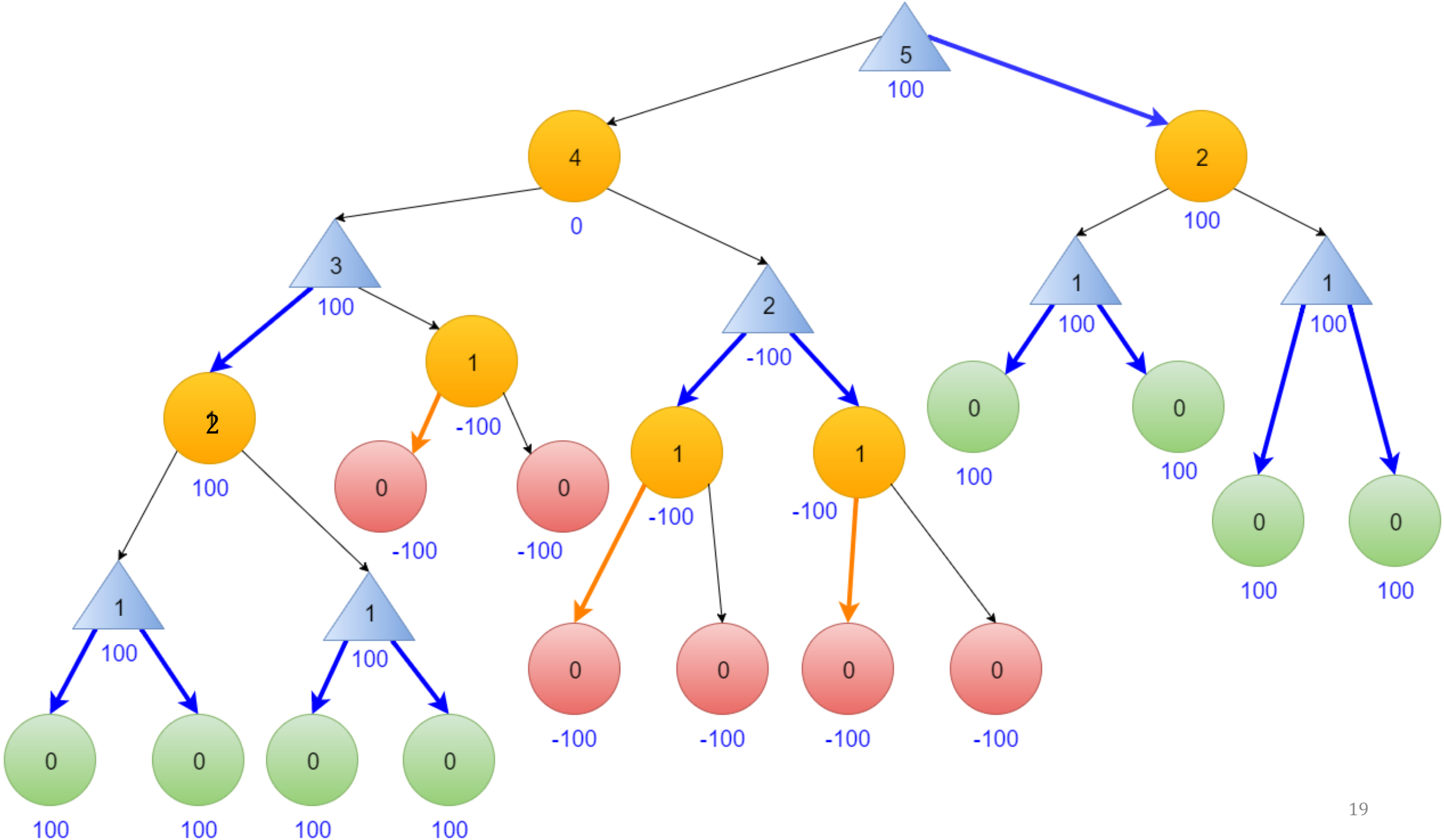

5
4
2
3
2
1
1
2
1
1
1
0
0
100
100
1
1
0
0
-100
-100
0
0
100
100
0
0
0
0
-100
-100
-100
-100
0
0
0
0
100
100
100
100


5
4
2
3
2
1
1
1
2
0
0
100
100
1
1
0
0
-100
-100
1
1
1
1
0
0
0
0
-100
-100
-100
-100
0
0
0
0
100
100
100
100


5
4
2
3
1
2
1
1
2
1
1
1
0
0
0
0
0
0
0
0
0
0
0
0
0
0
0
0
-100
-100
-100
-100
-100
-100
100
100
100
100
100
100
100
100

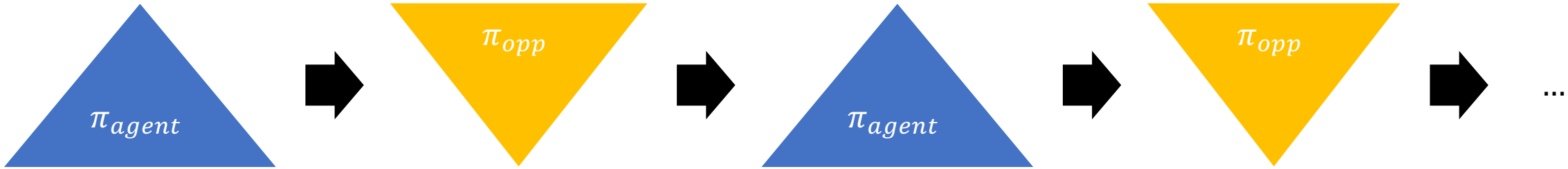

5
4
2
3
1
2
1
1
2
1
1
1
1
0
0
0
0
0
0
0
0
0
0
0
0
0
0
0
0
100
-100
-100
100
100
100
100
100
100
100
100
-100
-100
-100
-100


5
4
0
2
3
100
2
-100
1
1
1
-100
1
1
2
100
0
-100
0
-100
-100
-100
0
100
0
100
1
100
1
100
0
-100
0
-100
0
-100
0
-100
0
100
0
100
0
100
0
100
0
100
0
100

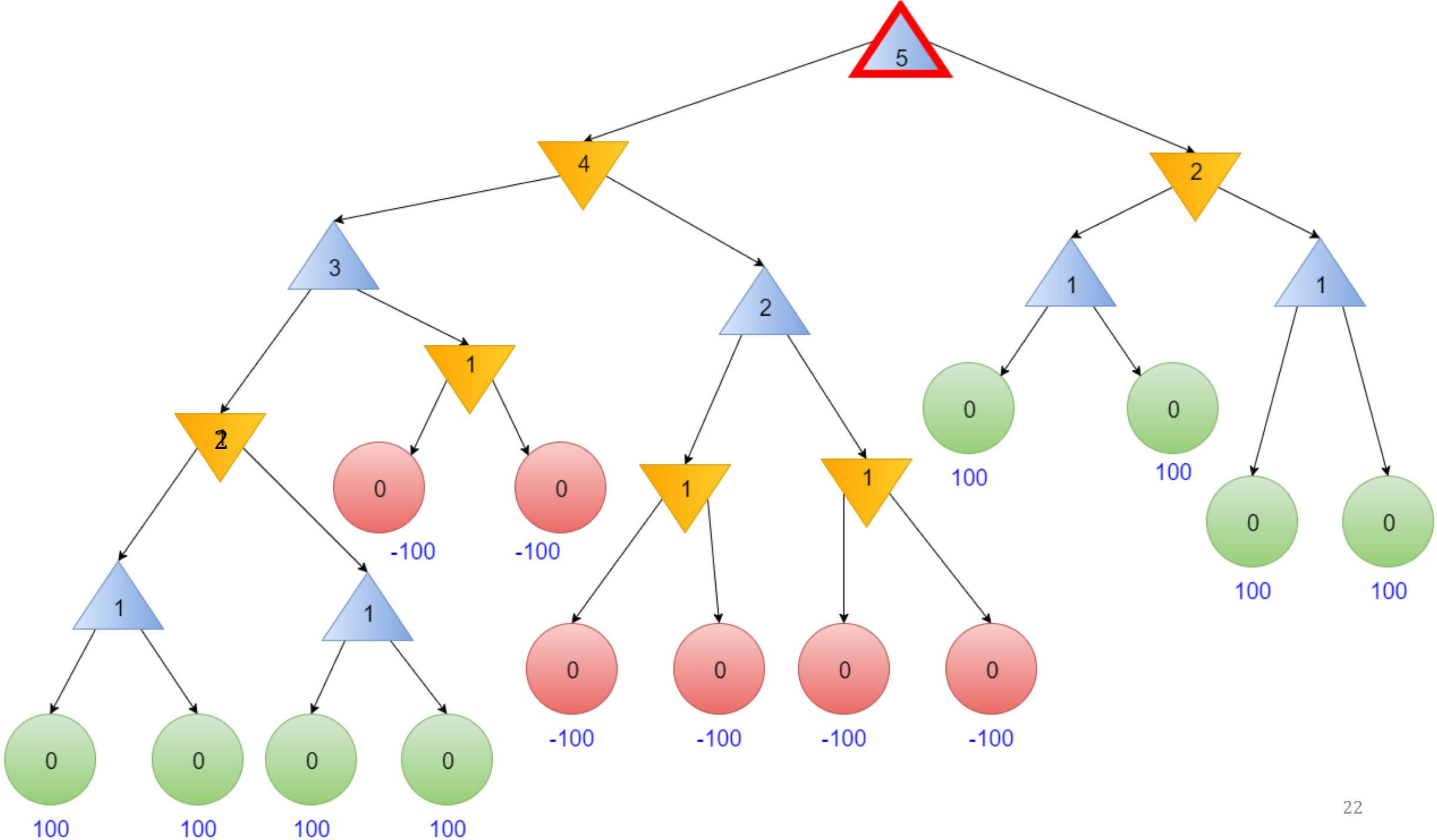

5
100
4
0
2
100
3
100
2
-100
1
100
1
100
1
-100
2
100
0
100
0
100
0
100
0
100
0
-100
0
-100
1
-100
1
-100
0
-100
0
-100
0
-100
0
-100
1
100
1
100
0
100
0
100
0
100
0
100

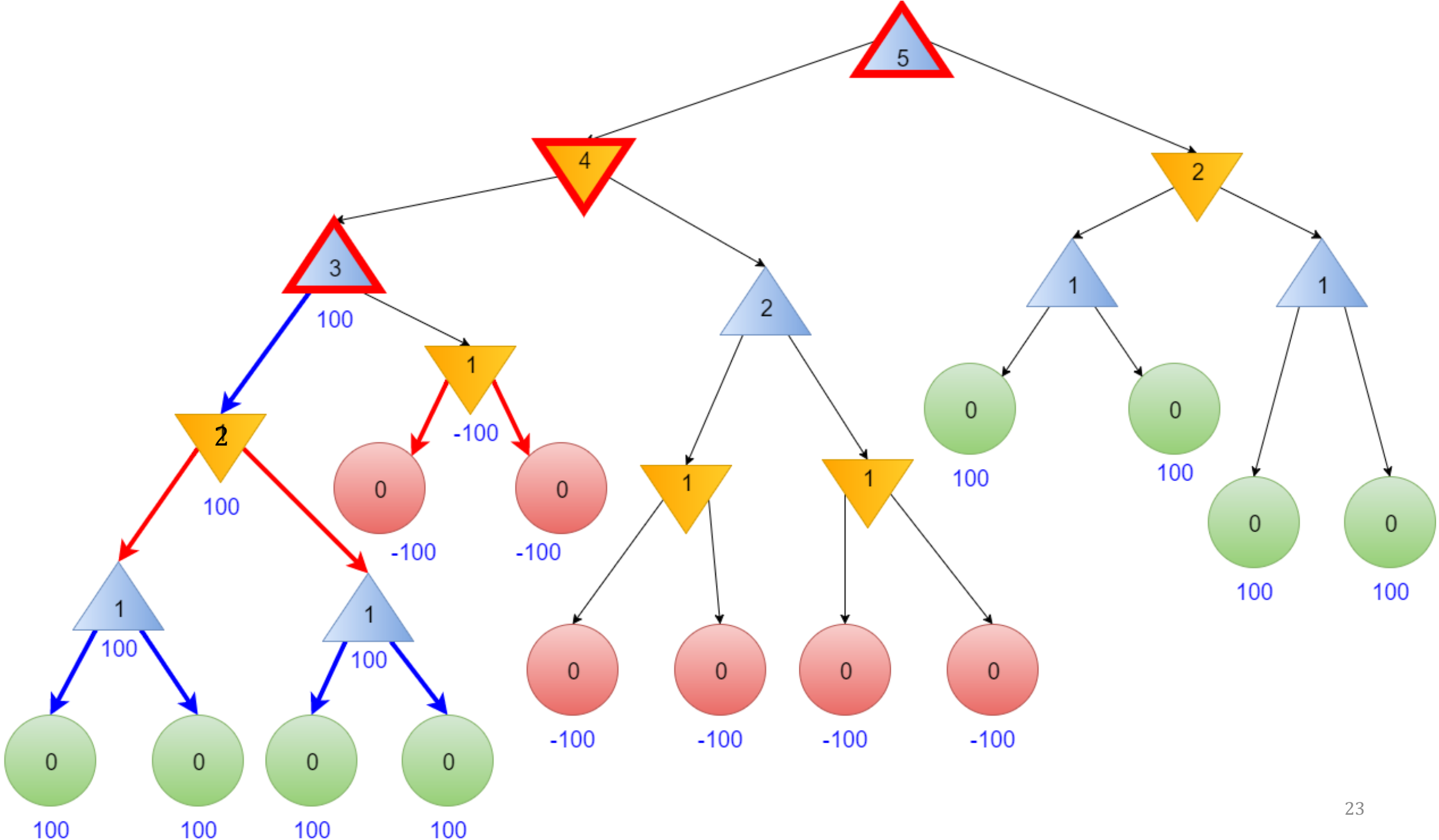
- Game Trees
- Expectimax
- Minimax

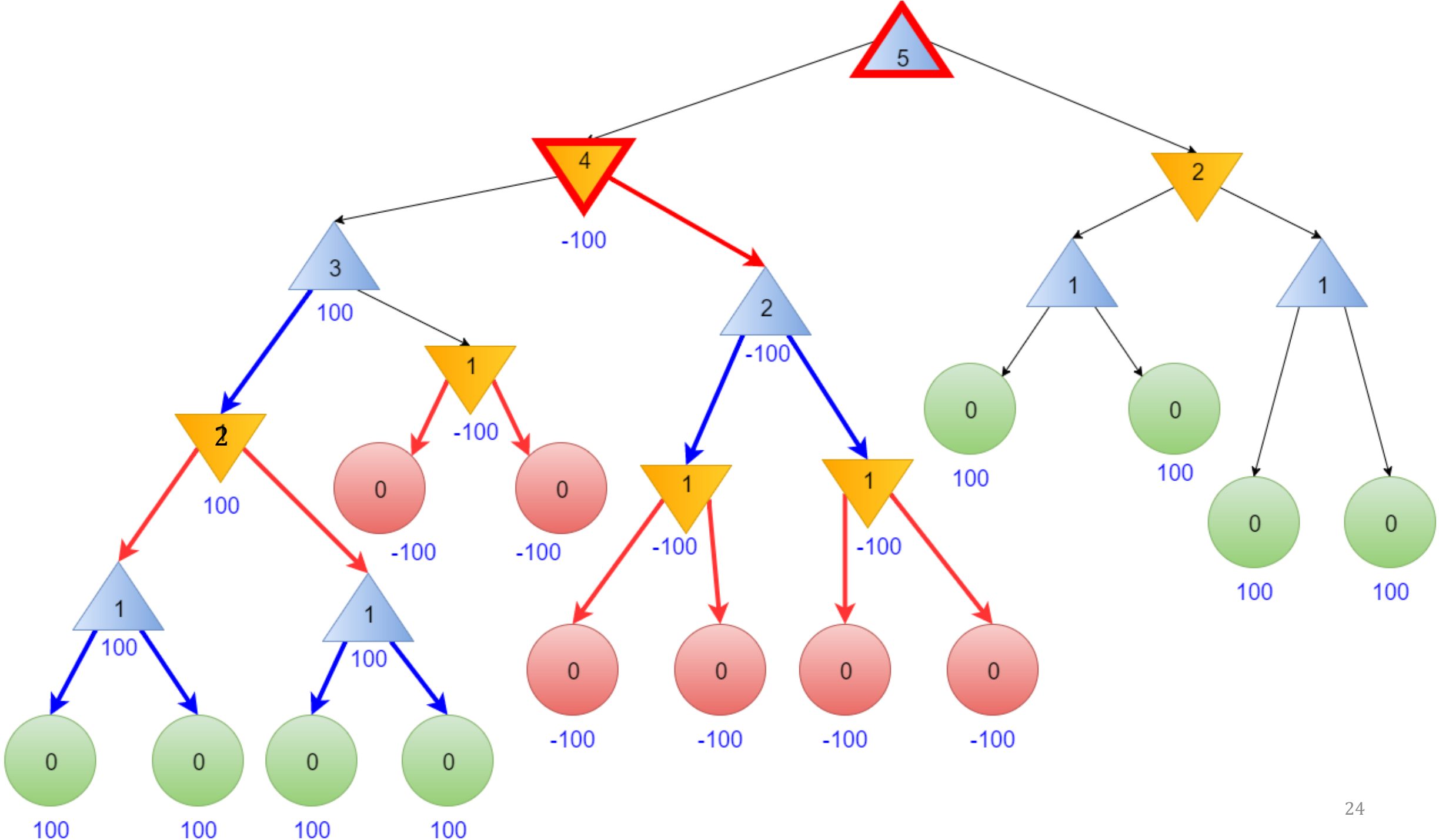
Minimax Recurrence

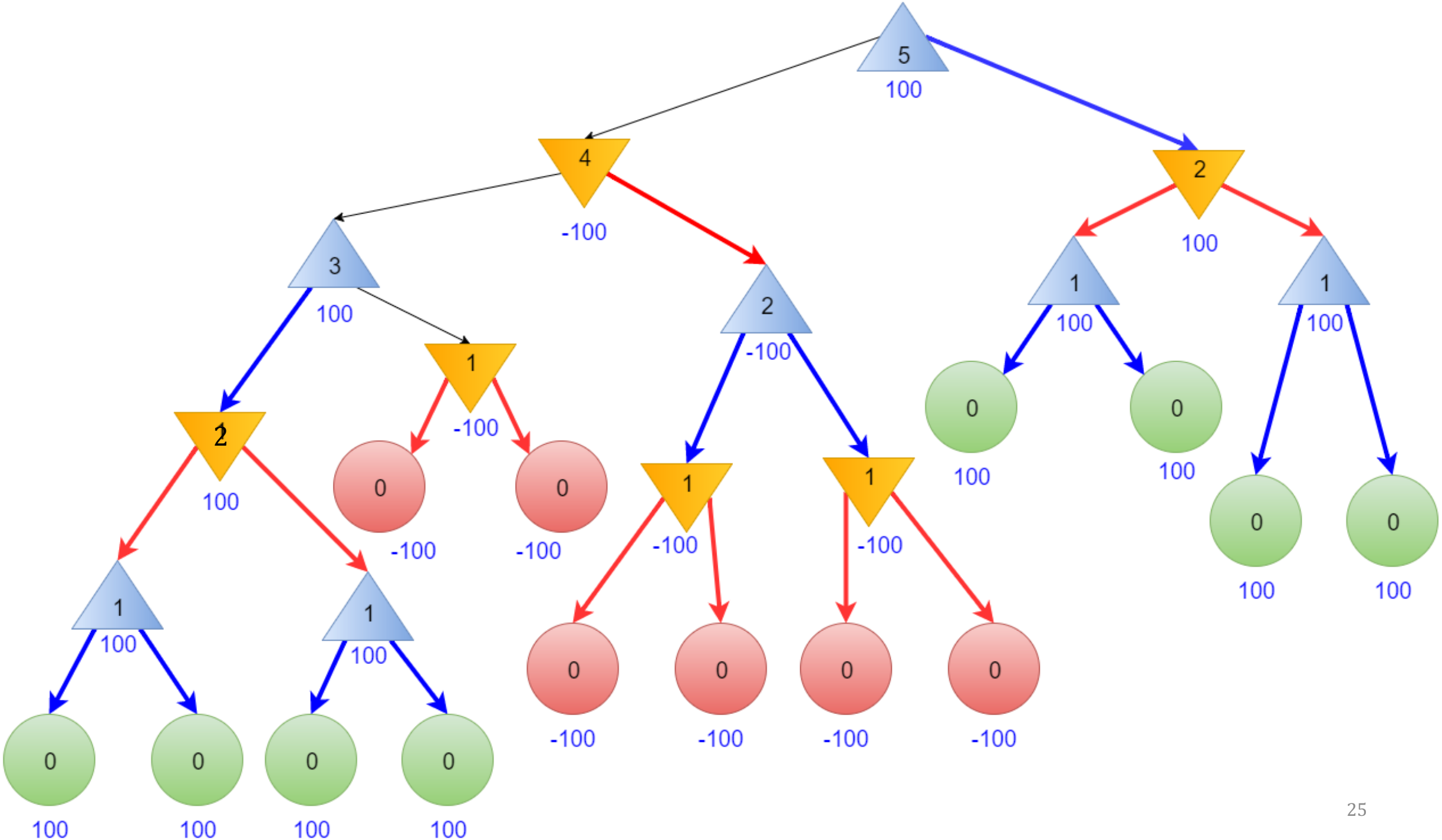


$$V_{opt}(s) = \begin{cases} \text{Utility}(s) & \text{IsEnd}(s) \\ \max_{a \in \text{Actions}(s)} V_{opt}(Succ(s,a)) & \text{Player}(s) = \text{agent} \\ \min_{a \in \text{Actions}(s)} V_{opt}(Succ(s,a)) & \text{Player}(s) = \text{opp} \end{cases}$$


5
4
2
3
2
1
1
2
0
0
0
0
-100
-100
1
1
1
1
0
0
0
0
0
0
0
0
100
100
100
100
-100
-100
-100
-100
1
1
0
0
0
0
100
100
100
100


5
4
2
3
100
1
2
100
-100
0
-100
0
-100
2
1
1
1
1
0
0
100
100
1
1
0
0
100
100
100
100
0
0
0
0
-100
-100
-100
-100
0
0
0
0
100
100
100
100


5
4
-100
2
3
100
1
-100
2
-100
1
100
0
-100
0
-100
2
-100
1
-100
1
-100
0
-100
0
-100
0
-100
0
-100
1
100
1
100
0
100
0
100
0
100
0
100
1
1
0
100
0
100
0
100
0
100


5
100
4
-100
2
100
3
100
1
-100
2
-100
1
100
1
100
1
100
2
100
1
100
1
100
1
100
1
100
0
-100
0
-100
0
-100
0
-100
0
-100
0
-100
0
-100
0
-100
0
100
0
100
0
100
0
100
0
100
0
100
0
100
0
100
0
100