CS221 Problem Workout Solutions

Nov 11

1) [CA session] Problem 1

Compute the conjunctive normal form (CNF) of the following two formulas and write every step of your computation:

(a) $\neg P \to \neg\neg(Q \vee (R \wedge \neg S))$

(b) $(P \to (Q \vee (R \wedge S))) \wedge (R \vee (S \to Q))$

Solution

(a)

$\neg P \to \neg\neg(Q \vee (R \wedge \neg S))$	Given
$\neg P \to (Q \vee (R \wedge \neg S))$	Double negation
$\neg\neg P \vee (Q \vee (R \wedge \neg S))$	Implication
$P \vee (Q \vee (R \wedge \neg S))$	Double negation
$(P \vee Q \vee R) \wedge (P \vee Q \vee \neg S)$	Distributivity

(b)

$(P \to (Q \vee (R \wedge S))) \wedge (R \vee (S \to Q))$	Given
$(P \to (Q \vee (R \wedge S))) \wedge (R \vee (\neg S \vee Q))$	Implication
$(\neg P \vee (Q \vee (R \wedge S))) \wedge (R \vee (\neg S \vee Q))$	Implication
$(\neg P \vee Q \vee R) \wedge (\neg P \vee Q \vee S) \wedge (R \vee \neg S \vee Q)$	Distributivity.

3) [Breakouts] Problem 3

Translate the following English sentences into first-order logic formulas:

(a) Every student takes at least one course.

(b) Every student who takes Analysis also takes Geometry.

(c) No student failed Chemistry but at least one student failed History.

Solution

(a)

$$\forall x\ (\text{Student}(x) \Rightarrow \exists y\ (\text{Course}(y) \wedge \text{Takes}(x,y)))$$

(b)

$$\forall x\ (\text{Student}(x) \wedge \text{Takes}(x,\text{Analysis}) \Rightarrow \text{Takes}(x,\text{Geometry}))$$

(c)

$$(\neg\exists s\,(\textit{Student}(s) \wedge \textit{Failed}(s,\ \textit{Chemistry}))) \wedge (\exists x\,(\textit{Student}(x) \wedge \textit{Failed}(x,\ \textit{History})))$$